On the Concept of Following Logically

Article *in* History and Philosophy of Logic · January 2002

CITATION
1

READS
189

2 authors, including:

Magda Stroinska
McMaster University
60 PUBLICATIONS **82** CITATIONS

SEE PROFILE

HISTORY AND PHILOSOPHY OF LOGIC, 23 (2002), 155–196

Taylor & Francis
Taylor & Francis Group

On the Concept of Following Logically

ALFRED TARSKI
TRANSLATED FROM THE POLISH AND GERMAN BY
MAGDA STROIŃSKA[1] AND DAVID HITCHCOCK[2]
[1]Department of Modern Languages and Linguistics, and [2]Department of Philosophy, McMaster University, 1280 Main Street West, Hamilton, Ontario L8S 4M2, Canada

Received February 2002 Revised July 2002

We provide for the first time an exact translation into English of the Polish version of Alfred Tarski's classic 1936 paper, whose title we translate as 'On the Concept of Following Logically'. We also provide in footnotes an exact translation of all respects in which the German version, used as the basis of the previously published and rather inexact English translation, differs from the Polish. Although the two versions are basically identical, to an extent that is even uncanny, we note more than 400 differences. Several dozen of these are substantive differences due to revisions by Tarski to the Polish version which he did not incorporate in the German version. With respect to these revisions the Polish version should be regarded as more authoritative than the German. Hence scholars limited to an English translation should use ours.

1. Translators' introduction

We offer below an exact translation of the Polish version of Alfred Tarski's classic 1936 paper, whose title we translate as 'On the concept of following logically', as well as of all variants in the German version. In this introduction Hitchcock argues that the Polish version is, in most places where there are substantive differences, more authoritative than the German version used as the basis of the only other published English translation (Tarski 1956, 1983, 409–23), which is rather inexact. Hence scholars should use our translation if they rely on an English version. In the final section of the introduction, Stroińska comments on Tarski's language. For reference purposes, and as a guide to anyone translating other writings of Tarski from Polish to English, we have prepared Polish–English and English–Polish glossaries recording the translations we used (see http://www.humanities.mcmaster.ca/~hitchckd/glossaries.htm).

2. Historical and theoretical importance of Tarski's paper

Tarski's paper is the origin of the generally accepted model-theoretic conception of what it is for a sentence in a formal language to follow logically from a set of sentences of the language (cf. Quine 1937; Church 1956, 325, n. 533; Beth 1969/1955, 38). According to Tarski's often-quoted definition, as we translate it below, '*We say that the sentence X follows logically from the sentences of the class* $\mathfrak{K}$ *if and only if every model of the class* $\mathfrak{K}$ *is at the same time a model of the sentence X*' (italics and spacing in original). In contemporary terminology, a sentence X in a formal language follows logically from a class $\mathfrak{K}$ of sentences if and only if every true interpretation of the sentences of the class $\mathfrak{K}$ is a true interpretation of the sentence X. This model-theoretic or semantic concept of following logically is generally used as a touchstone of the adequacy of a derivational system for a formal language; the formal system (language plus

History and Philosophy of Logic ISSN 0144-5340 print/ISSN 1464-5149 online © 2002 Taylor & Francis Ltd
http://www.tandf.co.uk/journals
DOI: 10.1080/01445340210000366833

derivational system) is taken to be 'sound' if and only if every sentence derivable from given sentences using the system follows logically from them in the model-theoretic sense, and taken to be 'complete' if and only if every sentence which follows logically from given sentences in the model-theoretic sense is derivable from them. (There are of course other criteria of completeness which antedate Tarski's paper, e.g. the criterion of the provability of all 'valid', i.e. logically true, formulae used by Post [1921] and Gödel [1930, 1931], or Post's [1921] criterion of provability of all formulae if any unprovable formula is added to the axioms of an axiomatically formulated logic.)

3. Status of the previous English translation

Especially since the publication of John Etchemendy's critique (1990) of Tarski's model-theoretic conception, there has been a substantial scholarly literature on Tarski's paper; see especially Sher (1991, 1996) for theoretical discussion and Gómez-Torrente (1996, 1998) for historical discussion. This literature uniformly cites the existing English translation of Tarski's paper (Tarski 1956, 1983, 409–20). But Tarski rather delicately expressed some dissatisfaction with the translations of this and other pre-war papers in his preface to the first edition (Tarski 1956, xi–xii) of the volume in which they appeared:

> In a few cases (in particular, in the case of the monograph on the concept of truth, which occupies nearly one-third of the present volume) the translation had to be based not upon the original, which was published in Polish, but upon the French or German version. This made it even harder for the translator to give a fully adequate rendering of the original intentions and ideas of the author. In addition, due to the factors of space and time, the translator was deprived of the benefit of extensively discussing with the author even the major difficulties encountered in his work, and so achieving a meeting of minds before the text was set up in type. To illustrate this point I may mention that, for various reasons, I have been unable so far to read a considerable part of the present text, and it seems more than likely that I shall not have read it before receiving a copy of the published book.

In his preface to the second edition, Tarski (1983, xiv) noted that he had made some corrections, but that a more thorough revision of the work, which might be desirable, was not feasible:

> ... new misprints and errors which have been noticed in the meantime have been corrected; some cross-references to other papers and references to later developments have been added; and certain changes have been carried through to clarify various passages of the earlier texts and, in particular, to remove a number of translation defects of the first edition. A more thorough and essential revision of the work (which might be desirable for several different reasons) was not feasible.

Tarski made only two corrections to the translation of the paper on following logically. He removed a duplicate printing of a footnote which appeared both on page 411 and on page 412. And he changed a misleading definite article on page

417 to an indefinite article, replacing 'we shall also call the model of the class L the *model of the sentence X*' by 'we shall also refer to a model of the class L as a *model of the sentence X*' (italics in original); it is quite clear from the context and from Tarski's other writings at the time (1936c, 1937) that Tarski was acutely aware that a sentence can have more than one model. The change suggests that Tarski did not have a chance to check the translation of the paper on following logically before it appeared in print in 1956, since he would very likely have made it in the first edition if he had had the opportunity. (An exact translation of the original German would read, 'we will call the model of the class L also a *model of the sentence X*' [italics in original]. The use of the definite article before the first occurrence of the word 'model' does not imply that a class has exactly one model; rather it refers back to a specific but arbitrary model of the class.)

Although the German and Polish versions of Tarski's paper, both written by him, bear an uncannily close resemblance, we have identified more than 400 differences between the two versions. In the vast majority of these cases, the 1956 translation follows the German rather than the Polish version of the paper. Since Polish was Tarski's native language, the choice of the German version as the basis of the translation was dubious. Dr Jan Tarski, Tarski's son and literary executor, notes in a letter dated 1 August 2000 that German was for his father in the 1930s '*the* second language, but nonetheless not a language of everyday discourse' (J. Tarski 2000). Further, as I shall argue below, in places where the Polish version differs substantively from the German version, the Polish version is universally better. Further, the 1956 translation is loose in places; for example, 'beschreiben' is translated as 'explain' rather than 'describe' (p. 413, n. 1), 'kann nicht ... zerstört werden' as 'cannot be affected' rather than 'cannot be destroyed' (p. 415), 'mit Hilfe' as 'in terms of' rather than 'with the help of' (p. 416), 'der Klasse L' as 'belonging to L' rather than 'of the class L' (p. 417), 'auf Grund dieser Begriffsbildung' as 'in terms of these concepts' rather than 'on the basis of this conceptual construction' (p. 417), and so on. Interested scholars can reconstruct an exact English translation of the German version from our notes, which translate exactly all its variant formulations.

4. Relation between the Polish and German versions

Tarski read the German version at an international congress of 'scientific philosophy' in Paris in September 1935; it was published the following year in the proceedings of the conference (Tarski 1936b). The Polish version of the paper (Tarski 1936a) was published in the first issue of the 1936 volume of the Polish journal *Przegląd Filozoficzny* (*Philosophical Review*). Tarski's bibliographical note to the English translation in the 1956 volume indicates that the Polish version was published first, but does not make clear which version was written first, or whether both were written more or less simultaneously:

> This is a summary of an address given ... in Paris, 1935. The article first appeared in print in Polish ..., and then in German ... (Tarski 1956, 1983, 409)

Tarski does not refer to either version as the original, as he does with respect to other articles in this collection. Jan Tarski comments as follows:

> As to the priority of composition of the two versions, my guess is that the two versions were written to some extent simultaneously. My father published several articles first in Polish and then in German, and so by 1936 he must have planned the double publication from the beginning. I surely think that in the case of a scientific article, it would be more natural to compose simultaneously than to translate a completed version into the other language. (J. Tarski 2000)

The suggestion of simultaneous preparation is confirmed by the uncanny basic resemblance between the two versions, even to the matching of constructions, metaphors, roots and derivative forms; the closeness of the two versions is more easily explained by someone's having written them at the same time in both languages, choosing a formulation which would work in either, than by someone's having written one version and then translated it into the other. It is also confirmed by the rather short time frame in which Tarski wrote the two versions. Carnap (1963, 61) reports that he persuaded Tarski during a visit to Vienna in June 1935 to read two papers at the September congress; Tarski delivered both papers in German at the congress, and published Polish versions of them in the first issue of the 1936 volume of *Przegląd Filozoficzny*. One can conjecture that, having decided to write two papers in German which communicated his ideas on the semantics of formalized languages to a philosophical audience, he asked the editor of the leading Polish philosophical journal (edited from the same city in which Tarski worked, Warsaw) whether the editor would be interested in a Polish version of the papers. Having received an expression of interest, Tarski could accomplish his task much more efficiently by writing both versions at once.

Most of the differences we have noted between the Polish and German versions can be explained by differences between ways of expressing the same point in the two languages, or by differences in the references, or by differences in the philosophical outlook of the intended audiences. A few of them appear to be the result of Tarski's having revised the Polish version before he submitted it to the editor of the Polish journal; he seems to have made at least some of these revisions after he returned from the September 1935 congress, in response to comments there. I discuss the more significant differences below.

5. Analytical outline of Tarski's paper

The following is an analytical outline of Tarski's paper, using the section numbers and headings which we have inserted in our translation.

0. Introduction: the difficulty of providing a precise definition for use in metamathematics of a concept already in use in everyday language. Compare Tarski (1956, 1983, 154–65).

1. The syntactic approach

1.1. Its initial success in deriving accepted mathematical theorems using a few simple rules of inference (e.g. detachment, substitution).

1.2. ω-incomplete deductive theories discovered (e.g. by Tarski in 1927).

1.3. Extensions to accommodate ω-incomplete theories (by Tarski?).

1.4. Failure: Gödel's proof that such attempts are doomed to fail. Need for a different approach for theoretical purposes, though the syntactic concept will remain important practically.

2. The semantic approach

2.1. Carnap's definition (1934): *The sentence X follows logically from the class of sentences* $\mathfrak{K}$ *if and only if the class consisting of all sentences of the class* $\mathfrak{K}$ *and of the negation of the sentence X is contradictory.* This definition is too closely tied to the specific properties of the particular languages for which it is given.

2.2. An alternative based on scientific semantics: Recent developments (by Tarski) make it possible for the first time to capture intuitions shared by many logicians in an exact and irreproachable form.

2.3. A two-part necessary condition of material adequacy for a definition of the circumstance that sentence X of a formalized language follows logically from the sentences of a class $\mathfrak{K}$: (1) From everyday intuitions: it cannot happen that all the sentences of the class $\mathfrak{K}$ are true but at the same time the sentence X is false. (2) From the formality of the relation: following cannot be lost as a result of replacing the names of objects in X and the sentences of $\mathfrak{K}$ by the names of other objects. These two conditions are expressed jointly in condition (*F*): *If in the sentences of the class* $\mathfrak{K}$ *and in the sentence X we replace the constant terms which are not general-logical terms correspondingly by arbitrary other constant terms (where we replace equiform constants everywhere by equiform constants) and in this way we obtain a new class of sentences* $\mathfrak{K}'$ *and a new sentence X', then the sentence X' must be true if only all sentences of the class* $\mathfrak{K}'$ *are true.* Defined terms should first be replaced by undefined terms, and each extra-logical constant should be replaced by an extra-logical constant of the same type.

2.4. Insufficiency of this necessary condition: Condition (*F*) can be satisfied in cases where X does not follow but the language does not have extra-logical constants designating the objects which would be a counter-example to the claim that it follows.

2.5. Preliminary definitions of semantic concepts needed for a formally correct and materially adequate definition of following logically for formalized languages (cf. Tarski 1933a, 1935, 1956, 1983, 152–278): *Satisfaction*: a relation between a sequence of objects and a sentential function (e.g. the sequence $<2, 3, 5>$ satisfies the function '$x+y=z$'), defined for each formalized language in the manner described in Tarski (1933a, 1935, 1956, 1983, 152–278). *Model* (of a class $\mathfrak{K}$ of sentences, of a sentence X) $=_{df}$ a sequence of objects which satisfies each sentential function obtained from the sentences of $\mathfrak{K}$ (the sentence X) by replacing all extra-logical constants in them (it) by corresponding variables: equiform constants by equiform variables and non-equiform constants by non-equiform variables.

2.6. Definition of following logically: *The sentence X follows logically from the sentences of the class* $\mathfrak{K}$ *if and only if every model of the sentences of the class* $\mathfrak{K}$ *is at the same time a model of the sentence X.*

2.7. Material adequacy of the definition: It conforms to everyday intuitions. It satisfies condition (1). It satisfies condition (2). Hence condition (*F*) is necessary for it. But condition (*F*) is not sufficient for it.

2.8. Relation to Carnap's definition: If we call a class of sentences or a sentence *contradictory* if it possesses no model, and we assume that for every sentence X in our formalized language the language contains its *negation* (a sentence whose models are just the sequences that are not models of X), then the two definitions (2.1, 2.6) are equivalent. Also, as Carnap intended, the *analytic* sentences (those for which every sequence of objects is a model) are just those which follow logically from every sentence of the language, and the contradictory sentences just those from which every sentence of the language follows.

3. Logical terms

3.1. An open question: Among the questions left open by the above construction, perhaps the most important is where to draw the dividing line between logical and extra-logical terms which lies at its basis. While some terms must clearly be regarded as logical if we are not to contradict everyday intuitions, we can extend the scope of logical terms beyond the usual without violating such intuitions, in the extreme extending it to all terms, thus making following formally coincide with following materially.

3.2. Philosophical implications: Wittgenstein and the Vienna Circle identify the concept of an analytic sentence with the concept of a tautology, a sentence which says nothing about the real world.

3.3. Prospects: If no objective arguments are found to justify the traditional dividing line between logical and extra-logical terms, concepts like following logically, analytic sentence and tautology will have to be treated as relative to a somewhat arbitrary division of the terms of a language into logical and extra-logical.

In what follows, I cite passages by section number or note letter, followed by sentence number (which we have assigned in accordance with the division of the Polish version into sentences). Thus '2.4.3' designates §2.4, sentence 3, which we translate as: 'Unfortunately, the matter does not seem so simple'. And 'E.1' designates note E, sentence 1, which we translate as: 'A clear-cut opposition of the two concepts under consideration is already contained in my article T_1, pp. 110 ff'.

6. Apparent revisions found in Polish but not in German

There are 11 differences between the two versions that cumulatively indicate that the Polish version is in many places a revision of the German version, and therefore in these respects more authoritative than it.

1. The Polish version has a number of phrases, clauses and even sentences lacking in the German version that clarify what is meant, provide a supporting reason, express a point more accurately or precisely, fill a lacuna or provide a link to the preceding discussion. In the rest of this paragraph, I cite 27 examples. (i) The task of accommodating the vagaries of the ordinary use of the concept of following is acknowledged in Polish but not in German to include that of capturing them as well as that of reconciling them (0.2). (ii) The ω-incomplete theory that Tarski produced in 1927 is called in Polish a 'deductive theory' but in German just a 'theory of that kind' (1.2.2). (iii) It is made clear in Polish but not in German that the particular sentences provable in the theory may be axioms or theorems rather than just theorems and that the respect in which they are similar is their form (1.2.2). (iv) The old syntactic concept of following is said in Polish to have been used 'in the construction of deductive theories' but in German to have been used 'by mathematical logicians' in some unspecified way (1.2.3). (v) A reason is given in Polish, but not in German, why new rules of inference like the rule of infinite induction cannot be reduced to the old rules: they make possible the proof of sentences previously impossible to prove (1.3.1). (vi) The connection of the rule of infinite induction with the previously cited example of an ω-incomplete theory is pointed out in Polish, but not in German (1.3.2). (vii) Whereas the German version notes merely that a proof of

infinitely many sentences does not in fact occur in practice, the Polish version correctly makes the stronger modal assertion that it cannot occur in practice (1.3.3). (viii) The Polish version alone states that the rule invoking a previous proof of sentence *B* is not precisely equivalent to the rule of infinite induction (1.3.4); the difference is apparently not in what can be proved using them, but only in that the rule of infinite induction operates within the object language, whereas the rule invoking a previous proof of sentence *B* would have to appeal in the metatheory of that language to what had been proved. (ix) The German version speaks of all the concepts and sentences of a theory's corresponding metatheory being interpreted in the theory; the Polish version speaks more cautiously, and probably more correctly, of all the concepts of the metatheory being so interpreted (1.3.6). (x) The question of whether the older syntactic rules of inference like detachment and substitution occupy a distinguished position is explicitly qualified in Polish, but not in German, by the phrase 'among all possible rules of inference' (1.3.10). (xi) The supposition that one could capture the content of the concept of following by supplementing previously recognized rules of inference is explicitly qualified in Polish, but not in German, as restricting itself to additional rules of a structural character (1.4.1). (xii) The Polish but not the German version makes explicit that the content of the concept of following has by no means been exhausted by the rules used until now (1.4.1). (xiii) Only the Polish version makes explicit that the syntactic concept of following which will probably continue to be used in constructing deductive theories may be widened with the help of new rules of inference (1.4.4). (xiv) Tarski describes his task in Polish as that of constructing a (formally) correct and (materially) adequate definition of the concept of following for formalized languages, thus making clear that his approach is parallel to that in his monograph on truth (Tarski 1933a, 39–40, 1935, 305–6, 1956, 1983, 187–8; cf. 1944, 341–2), whereas in German he merely describes his task as that of providing a '(materially) adequate' definition (2.2.1, 3.1.1). (xv) Similarly, Tarski says in Polish that the term 'true' can be defined 'correctly and adequately', whereas in German he uses the confusing phrase 'exactly and materially correctly' (2.4.2). (xvi) Tarski claims in Polish, but not in German, that the methods of scientific semantics make it possible to capture our everyday intuitions about the concept of following in a way which is not subject to reproach (2.2.3). (xvii) Only the Polish version makes explicit that the sentence *X* under consideration is an arbitrary sentence that follows from the sentences of the class $\mathfrak{K}$ (2.3.2). (xviii) Condition (*F*) of material adequacy for a concept of following logically for formalized languages is described in Polish more accurately as expressing jointly the conditions of necessary truth-preservation and independence of knowledge of objects designated by extra-logical constants, rather than as combining these conditions (2.3.5). (xix) In the statement of condition (*F*) Tarski uses in Polish the same expression 'general-logical' to characterize logical terms as he used in his monograph on truth (Tarski 1933a, 21, 23, 1935, 285, 287; 1956, 1983, 170, 172) and his paper on the concepts of ω-consistency and ω-completeness (Tarski 1933b, 99); Tarski's regular contrast during this period of 'general-logical' terms or concepts with 'specifically metalinguistic' (1933a, 21, 1935, 287, 1956, 1983, 172) or 'specifically structural-descriptive'

(1933b, 99, 1956, 1983, 281) or 'specifically mathematical' (1936c, 24, 1937, 12, 1946/1941, 18) terms or concepts seems to reflect a view that logical terms and concepts are distinguished from extra-logical ones by their occurrence in all fields of science and in everyday life (cf. 1936c, 24, 1937, 12, 1946/1941, 18), a view later superseded by Tarski's more principled criterion of invariance under all transformations of a universe of discourse into itself (Tarski 1986/1966, 149). In the German version of the present paper Tarski uses the less meaningful expression 'purely logical' (2.3.5). (xx) Only the Polish version makes explicit from the outset that the condition that names of all possible objects occur in a language is an assumption (2.4.5). (xxi) It is only explicit in Polish that it is more exact to refer to satisfaction of a sentential function by a sequence of objects than by individual objects (2.5.2). (xxii) Whereas in German the transformation of the sentences of a class into the sentential function required for the construction of a model is said to be subject to the requirement that 'equal' constants are to be replaced by equal variables and 'different' constants by different variables, the Polish version specifies how they are to be equal or different: in form (2.5.8). (xxiii) Tarski uses a conditional in German and a biconditional in Polish to express the relation, when all terms of the language are treated as logical, between the condition that either the sentence X is true or a sentence of the class $\mathfrak{K}$ is false (i.e. following materially or 'material implication') and the circumstance that X follows logically in the defined sense from the sentences of $\mathfrak{K}$; the biconditional expresses more accurately his claim that in this case the concept of following logically would coincide with the concept of following materially (3.1.7). I defend Tarski's claim against contemporary sceptics in §8 below. (xxiv) A condition for realizing the importance of the question of distinguishing logical from extra-logical terms for certain general philosophical views is plausibly presented in Polish as necessary, but implausibly in German as sufficient (3.2.1). (xxv) The division of terms into logical and extra-logical is correctly said in Polish to influence the definition of the term 'contradictory' as well as of the term 'analytic', whereas only its influence on the definition of the term 'analytic' is mentioned in German (3.2.1). (xxvi) In the final paragraph, only the Polish version makes explicit that the division of terms into logical and extra-logical is relative to a particular language (3.3.2). (xxvii) In the second paragraph of note E, the Polish version gives a more complete description of Carnap's contribution in his *Logical Syntax of Language* by mentioning Carnap's definitions of the concept of following for certain concrete deductive theories (E.5). (xxviii) Although there are a few cases where the German version includes qualifiers not present in the Polish (e.g. 1.2.3, 1.3.1, 2.3.5, 2.5.9, 2.8.2), these additional qualifiers generally do not improve the sense. It is possible of course that Tarski also made revisions to the German that he did not make to a previously identical Polish version.

2. Whereas the Polish version speaks of 'operations' on axioms or theorems when rules of inference are applied, the German version calls them 'transformations' (1.1.4, 1.1.5). Since the word 'transformation' could be taken to imply that a rule of inference has only one input sentence, the Polish version is less misleading.

3. Tarski produces in one sentence (2.3.4) a highly complex chain of reasoning for the second of his conditions of material adequacy, that the relation of following logically cannot be lost as a result of replacing in the sentences among which it obtains names of objects talked about in those sentences by names of other objects. The punctuation of the German version indicates a different structure of argumentation than the punctuation of the Polish version. The structure indicated by the Polish is more plausible, as can be seen by setting out the structure of argumentation in the style of an annotated deduction in a natural deduction system. The punctuation and content of the Polish indicate the following structure:

   1. It is a question here of the relation of following logically. (assumption)
   2. It is a question here of the relation of following formally. (from 1)
   3. It is a question here of a relation which has to be completely determined by the form of the sentences among which it obtains. (from 2)
   4. Following cannot depend on our knowledge of the external world. (from 3)
   5. Following cannot depend on our knowledge of the objects which are spoken about in the sentences of the class $\mathfrak{K}$ or in the sentence X. (from 4, by instantiation)
   6. Following cannot be lost as a result of our replacing the names of these objects in the sentences under consideration by names of other objects. (from 5 [or 3?])

   The punctuation and content of the German indicate the following structure:

   1. It is a question here of the concept of following logically. (assumption)
   2. It is a question here of the concept of following formally. (from 1)
   3. It is a question here of a relation which has to be uniquely determined by the form of the sentences among which it obtains. (from 2)
   4. The relation of following cannot be destroyed as a result of one replacing the designations of the objects mentioned in the sentences under consideration by designations of some other objects. (from 3)
   5. This relation can in no way be influenced by empirical knowledge. (from 4)
   6. This relation can in no way be influenced by the knowledge of the objects which are spoken about in the sentences of the class $\mathfrak{K}$ or in the sentence X. (from 5, by instantiation)

   In the German version, the inference from 3 to 4, signalled by a colon, is highly implausible.

4. Generally Tarski writes of a sentence X as following from 'the sentences of a class $\mathfrak{K}$', rather than from 'the class $\mathfrak{K}$ of sentences'. But he speaks of the sentence X as following from the class $\mathfrak{K}$ in a few places, one in Polish only (2.1.2), three in German only (2.4.1, 2.4.5, 2.7.3), one in both versions (2.8.5, where the class is empty, so that he is forced to speak of X as following from the class). Since the definiens in Tarski's model-theoretic definition talks about the class $\mathfrak{K}$ rather than its constituent sentences, there is no theoretical difference between the two ways of formulating the relatum of the relation of following logically. But the greater consistency in the Polish

version suggests an attempt to have the paper speak uniformly of a sentence as following from the sentences of a class, rather than from the class.

5. The Polish version is more cautious in several respects than the German version in speaking about the views of Carnap. At 2.8.1 Tarski in German brings his proposed definition of following logically 'into accord with' that of Carnap, but in Polish brings it only 'closer to' Carnap's definition. At 2.8.5 definitions of analytic and contradictory sentences in terms of Tarski's proposed definition of following logically are said in German to be 'just as for Carnap' but in Polish only to be 'in accord with Carnap's intentions'. In note F, Tarski writes in German that another definition by Carnap of following adapted to a formalized language with a very simple structure cannot be carried over to languages of more complicated structure, but in Polish that it 'cannot be extended in a natural way' (F.2) to other less elementary languages. In note H, after an attack on Carnap's attempt to define the concept of following logically on the basis of his so-called 'general syntax', an attack based on its dependence on the richness of the language under consideration, the Polish version softens the blow by noting, in a sentence (H.3) absent from the German, that this attempt by Carnap is not closely connected to the present considerations. The differences in the treatment of Carnap's views seem best explained as changes to the Polish version made in response to comments by Carnap, who attended the Paris conference (Carnap 1963, 61).

6. At 3.2.1 the German version says that the concept of a tautology as a sentence that 'says nothing about reality' was of fundamental significance for the whole Vienna Circle, whereas the Polish version says that the concept played and still plays a prominent role for 'almost' the whole Vienna Circle. The qualification 'almost' is likely to have been an accommodation of some member of the Vienna Circle present at the Paris conference, and the softening of the characterization of the importance of the concept of tautology for the Vienna Circle may also have been a response to comments there by its members. Carnap (1963, 61) reports that several members and supporters of the Vienna Circle, including Otto Neurath and Arne Ness, expressed 'vehement opposition' at the conference to Tarski's ideas. Some inkling of these objections can be gathered from the second part of Tarski's 'The semantic conception of truth and the foundations of semantics', entitled 'Polemical remarks' (Tarski 1944, 355–70, esp. 362–4).

7. When he explains the concept of an ω-incomplete theory, Tarski speaks in German of 'the given property P' but in Polish simply of 'the property P' (1.2.2). Since no property has been previously mentioned, the expression in Polish is more felicitous.

8. In German, Tarski sometimes (2.1.3, 3.2.1, 3.3.2, K.1) puts quotation marks around the names of concepts to which he is referring, whereas in Polish he consistently uses such names without quotation marks. Since he does not regard concepts as linguistic entities, the Polish version is correct in this respect and the German version mistaken. (The quotation marks in both

Polish and German around a lengthy phrase used at 1.3.8 to describe a concept are needed to make clear the boundaries of the description.)

9. The German version rather confusingly uses the same italic font for letters designating sentences of an object language (e.g. *X*) and for letters designating classes of such sentences (e.g. *K*, *L*) or classes of sentential functions (e.g. *L'*). The Polish version uses italics for letters designating sentences of an object language (e.g. *X*) but Gothic type for letters designating classes of such sentences (e.g. $\mathfrak{K}$, $\mathfrak{L}$) or classes of sentential functions (e.g. $\mathfrak{L}'$). It is hard to imagine that a mathematician as scrupulous about exactness as Tarski would have used the same symbols for a class and for its members; good mathematicians do not confuse their readers in that way, and it appears from sampling that none of Tarski's other published papers do so. One possible explanation of the German version's uniformity of font for names of sets and of their members is that Tarski had no opportunity to proof-read the German version before it was published.

10. We detected only one misprint in the Polish version (*ogólne* for *ogólnej* at H.3), but several in the German version: *nur*, 'only', for *nun*, 'now' (1.1.5), *w.s.w.* for *u.s.w.* and *im* for *in* (1.3.2), *Ubergang* for *Übergang* (1.3.5), *Gegenständ* for *Gegenstand* (2.1.1), *äusseren* for *äussersten* (3.1.7), *muss* for *müssen* (3.3.2), *Uber* for *Über* (C.1), *zulässlich* for *zulässig* (E.3). The greater number of misprints in German adds additional support to the supposition that Tarski had no opportunity to check the proofs of the German version.

11. The Polish version alone has a note at the end of the paper (before note A) stating when and where Tarski delivered the report. While the absence of the note from the German is easily explained by the fact that the German version appears in the proceedings of the conference where Tarski read his paper, its presence in the Polish suggests that Tarski wrote at least this note after he returned from the Paris congress in September 1935. He may well have revised the Polish paper in other respects at the same time.

7. Other substantive differences between the Polish and German versions

There are a few other substantive difference between the Polish paper and the German paper. Though apparently irrelevant to the question of the order of composition, they deserve comment.

1. In Polish Tarski talks about the 'everyday' usage and content of the concept of following, the 'everyday concept' of following and its 'everyday sense', whereas in German he talks correspondingly about the concept's 'common' usage and content, the 'common concept' of following and its 'common sense;' cf. 0.1, 1.1.1, 1.1.5, 1.2.3, 1.4.2, 1.4.3, 2.7.1, 3.3.2. He also appeals in Polish, but not in German, to 'everyday intuitions' (1.2.3, 2.3.3, 2.4.4, 3.1.4, 3.1.5). He does however twice refer in the German version to 'everyday language' (0.1, 0.2), making clear there that by the common usage of the concept of following he means its usage in everyday language. The Polish terminology perhaps makes more clear than the German terminology that Tarski uses as his

touchstone everyday speech, not the inferential habits of mathematicians (as some contemporary commentators have assumed); for the relevance of this fact to Etchemendy's attack on Tarski's definition, see §8 below. On the other hand, the word 'common' more obviously contrasts with the word 'proper' used in both Polish and German for the concept of following which should be used in the methodology of the deductive sciences.

2. Apart from the systematic difference just mentioned between the 'everyday concept' of following in Polish and the 'common concept' in German, Tarski occasionally varies unsystematically the phrases he uses to refer to the concept of following: 'concept' in Polish for 'common concept' (0.2) or for 'proper concept' (2.3.5) in German, 'concept of following logically' in Polish for 'concept of following' in German (2.7.4), 'following' in Polish for 'the concept of following' in German (3.1.4, K.6,), 'following logically' in Polish for 'the concept of following logically' in German (2.6.1, F.3), 'following formally' in Polish for 'the concept of following formally' in German (H.1), 'the relation of following logically, i.e. *formally*' in Polish for 'the concept of following logically, i.e. *formally*' in German (2.3.4), 'following' in Polish for 'this relation' in German (2.3.4). These variations appear to be inconsequential and accidental.

3. In indicating that each particular sentence of the form '*n* possesses the property *P*' is provable in a given ω-incomplete theory, Tarski gives one more particular instance in Polish than he does in German (1.2.2). The discrepancy seems inconsequential, and difficult to explain.

4. Tarski uses the word 'intuitions' extensively (nine times) in the Polish version, but never in the German version. The Polish speaks about 'intuitions' connected with the use of the concept of following, the German of 'tendencies' (0.2, 2.2.2, 2.2.3). The Polish speaks about what seems certain 'from the point of view of everyday intuitions', the German of what 'intuitively' seems certain (1.2.3). (The German word here, *inhaltlich*, literally means 'with respect to content' and could be translated 'conceptually'; Tarski uses *inhaltlich* in one other place, in the immediately following sentence 1.3.1, where we follow the lead of the Polish in translating it as 'intuitively'.) The Polish speaks about what is clear 'from the point of view of everyday intuitions', the German of what is clear from the 'intuitive standpoint' (2.3.3). The Polish speaks about what does not follow 'from the point of view of everyday intuitions', the German speaks of what does not follow 'in the common sense' (2.4.4). The Polish speaks of capturing or contradicting 'intuitions' manifested in everyday usage, the German of fitting with or contradicting everyday or common 'linguistic usage' (2.7.1, 3.1.4, 3.1.6). Tarski may have changed his terminology to suit his philosophical audience: at the congress of 'scientific philosophy' in Paris his audience would have consisted of logical positivists and their sympathizers ready to accept observations about patterns of linguistic usage as legitimate but not so receptive to talk of 'intuitions', whereas the readers of the Polish philosophy journal perhaps included many phenomenologists who conceived the task of philosophy as that of arriving at intuitions of essences.

5. In Polish, Tarski says that the relation of following logically cannot depend on our knowledge of the external world, in German that it cannot in any way be influenced by empirical knowledge (2.3.4). Talk about 'the external world' was common at the time in Polish phenomenology and is traditional in early modern philosophy, whereas logical positivists would be more accustomed to talking about 'empirical knowledge'. Here too Tarski appears to be tailoring his vocabulary to his audience.

8. Comments on the content of Tarski's paper

This is not the place for a full historical and theoretical discussion of Tarski's paper; for such discussion, readers can consult the aforementioned writings by Sher (1991, 1996) and Gómez-Torrente (1996, 1998). But a few remarks are perhaps appropriate to orient the reader.

Although it is the ancestor of the contemporary model-theoretic conception, the concept of following logically which Tarski defines in this paper is not the same as the contemporary one. Nowadays logicians work with formal languages, whose extra-logical constants are uninterpreted (until an interpretation is fixed in a particular case). To provide an interpretation or model of such a language, one typically specifies a domain or universe of discourse (typically a non-empty set of objects) and assigns to each individual constant a unique object in the domain, to each monadic first-order predicate a subset of the domain, to each dyadic first-order predicate a subset of the set of ordered pairs of objects in the domain, and so forth. Tarski however worked with what he called *formalized* languages, in which the extra-logical constants are interpreted and the domain is fixed. For example, in his monograph on truth, 'The concept of truth in formalized languages' (1933a, 1935, 1956, 1983, 152–278), Tarski constructs his definition of truth initially with reference to a formalized metalanguage for the calculus of classes, in which the extra-logical constants are names of the concrete signs or expressions of the language of the calculus of classes; different classes of variables in the metalanguage range respectively over classes of individuals, sequences of classes of individuals, expressions, sequences of expressions, classes of expressions, natural numbers and sequences of natural numbers (Tarski 1956, 1983, 172–3). The use of formalized rather than formal languages explains the rather cumbersome definition of a model in the paper on following logically. Because he uses a formalized language with interpreted extra-logical constants, Tarski must first replace extra-logical constants by variables of the same type and then consider what sequences of objects satisfy the resulting sentential function.

Further, the concept of a model in Tarski's 1936 paper is not the contemporary concept. In contemporary mathematics, as Hodges (1986) points out, a model or structure is roughly a collection of elements with labelled relations defined on them. A sequence of objects is not such a structure.

The letter 'F' in Tarski's condition (*F*) of material adequacy stands for 'following', not for 'formality', as Gómez-Torrente (1996) conjectured. Tarski uses the letter 'F' in the German version, standing for 'Folgerung', and the letter 'W' in the Polish version, standing for 'wynikanie', the word abbreviated corresponding in each case to the English 'following'. If the word is translated 'consequence', the letter 'C' should be used instead. Tarski's practice here corresponds to his abbreviation in his monograph on truth of the condition of material adequacy for a definition of truth as convention 'T' for truth (Tarski 1956, 1983, 187–8; cf. Tarski 1944, 344); in

German (Tarski 1935, 305–6) he uses the letter 'W' (for 'Wahrheit') and in Polish (Tarski 1933a, 40) the letter 'P' (for 'prawda'). In each case, the letter abbreviates the name of the concept whose condition of material adequacy is being provided, not some feature of that condition itself.

John Etchemendy (1990) advanced what he took to be an internal objection to Tarski's model-theoretic conception, that it fails to capture the modal feature of the intuitive concept of following logically. According to the intuitive concept, as Etchemendy (following Tarski) understands it, a sentence X follows from given sentences if and only if it is impossible for the given sentences to be all true and the sentence X false. (Tarski's argument at the end of 1.2.3 assumes that necessary truth-preservation is a sufficient condition for the everyday concept of following to be exemplified. But he adds the additional requirement of independence of extra-logical constants in his condition (F), thus assuming a more restrictive condition of material adequacy than mere conformity to the everyday or intuitive concept of following.) Etchemendy argued that Tarski's definition both undergenerates and overgenerates instances of following logically by comparison to the intuitive concept.

Addressing Etchemendy's critique requires, among other things, some clarification of the modal operator in the intuitive condition of necessary truth-preservation. Etchemendy charges Tarski with committing (at 2.7.3) 'Tarski's fallacy': inferring from the necessary truth of a conditional to the necessary truth of its consequent given its unmodalized antecedent ($\Box[P \rightarrow Q] \therefore P \rightarrow \Box Q$, where P is the definiens of Tarski's definition and $\Box Q$ is the intuitively based condition of material adequacy: necessary truth-preservation). Since it is *prima facie* unlikely that a logician of Tarski's stature would have committed such an elementary blunder at the height of his career, and that he would still be unaware of it more than 40 years later when editing the second edition of this paper's English translation, there is some burden on the interpreter of Tarski's paper to find a more charitable interpretation than Etchemendy's. Unfortunately, Tarski nowhere explains the meaning of the modal words 'must' and 'impossible' which he uses in formulating the intuitive condition for following logically; in view of Tarski's avoidance of such modal operators in the formalized languages that he investigated, his reticence is perhaps understandable. The necessity he has in mind is unlikely to be a necessity of the strong 'logical' sort envisaged by Etchemendy, which is compatible with even mathematically false but 'logically' possible claims, such as the claim that there are only finitely many objects. For in this very paper Tarski expresses scepticism about the concept of a sentence which says nothing about the real world (3.2); thus, he appears to have regarded even 'analytic' or logically true sentences as saying something about the world and thus to have regarded the necessary truth of a logically true sentence as compatible with its saying something about the world. A few years later, in 1940, Tarski agreed with Quine against Carnap that the distinction between logical and factual truth is at best a matter of degree (Carnap 1963, 64). And a few years after that, in 1944, Tarski described himself as inclined to believe that 'logical and mathematical truths don't differ in their origin from empirical truths—both are results of accumulated experience ... . I think that I am ready to reject certain logical premises (axioms) of our science in exactly the same circumstances in which I am ready to reject empirical premises (e.g. physical hypotheses) .... I can imagine that certain experiences of a very fundamental nature may make us inclined to change just some axioms of logic' (1987/1944, 30–1). Although there is some tension

between this strain in Tarski's thought and his claim in the present paper (at 2.3.4) that the relation of following logically is independent of our knowledge of the external world (or, as the German has it, of empirical knowledge), Tarski derives this latter independence claim from the formality of the relation, not from the intuitively based condition of necessary truth-preservation. In other words, he envisages the possibility that a sentence might follow in the intuitive sense, though not formally, in virtue of our knowledge of the external world. For example, from the point of view of everyday intuitions, the sentence 'object a falls toward the surface of the earth with an acceleration of 9.8 ms^{-2}' follows from the sentences 'object a is at the top of a vertical vacuum tube near the surface of the earth' and 'no electromagnetic forces act on object a': if the last two sentences are both true, then the first one must also be true. The relation of necessary truth-preservation in this case is grounded on a complex body of knowledge about the external world, including not only Newton's law of universal gravitation but also information about the mass of the earth and the relative masses and distances from object a of other objects that exercise a gravitational attraction on it. It is certainly not preserved for every uniform substitution on the extra-logical constants in the sentences. The necessity involved in this instance is clearly not the sort of logical necessity which Etchemendy attributes to Tarski.

The above argument is reinforced by the repeated use of 'everyday' rather than 'common' in the Polish version of Tarski's paper, a use which makes more clear (as mentioned in §7.1 above) that the first condition of material adequacy proposed by Tarski was based on everyday usage of the word 'follows', not on the inferential habits of mathematicians. Thus the necessity in this condition of necessary truth-preservation is *prima facie* unlikely to be a strongly logical necessity. In this one respect at least, our exact translation of the Polish version provides additional defence of Tarski's paper against Etchemendy's critique.

Sher (1996, 679) defends a construal of 'impossible' in the intuitive condition for following logically as 'incompatible with the formal structure specified by the logical terms', where a logical term is invariant under isomorphic structures; Tarski likewise (some 30 years after publication of the present paper) defined a logical notion as a notion 'invariant under all possible one-one transformations of the world [i.e. universe of discourse—DH] onto itself' (1986/1966, 149), but clearly had not worked out this conception at the time of his 1936 paper, given his linguistic echo in his condition (*F*) of his earlier characterization (1933a, 21, 1935, 285, 1956, 1983, 170) of logical terms as those found generally in any system of mathematical logic as opposed to those specific to only some mathematical theories (a criterion that led him to count the symbol for class inclusion as a logical term) and his concluding remarks in this paper (3.3) about the possible relativity of the concept of a logical term.

Gómez-Torrente (1998) construes the modal expressions in the intuitively based condition and in condition (*F*) as merely signs of generality, which could be eliminated without loss of meaning; this interpretation however makes the condition of necessary truth-preservation duplicate the formality condition, a duplication which it would be hard to imagine Tarski failing to note.

A fourth interpretation can perhaps be derived from Tarski's later association of this sense of 'must' with infallibility: 'Intuitively all the rules of proof [in a formalized mathematical theory—DH] appear to be infallible, in the sense that a sentence which is directly derivable from true sentences by means of any of these

rules must be true itself' (Tarski 1969, 293). In the present paper Tarski refers twice to the intuitive infallibility of rules of inference (1.3.1, 1.3.7), which he explains (1.3.1) as always leading from true sentences to other true sentences. The 'always' in this explanation seems to mean 'no matter what true sentences we apply the rule to'. Tarski uses in Polish the same metaphor of temporal universality in justifying his claim at 1.2.3 that, from the point of view of everyday intuitions, a universal sentence follows from a set of particular sentences: '*whenever* all these sentences are true, then also the sentence A must be true' (italics added), and also in German in his statement of the condition for a sentence to follow logically which holds from an intuitive standpoint: 'it can *never* happen that the class $\mathfrak{K}$ consists of nothing but true sentences but *at the same time* the sentence X is false' (italics added). Note too the use in both Polish and German of a temporal metaphor in the definition of following logically: 'every model of the class $\mathfrak{K}$ is *at the same time* a model of the sentence X' (italics added). Since we are dealing in these three passages with definite sentences, rather than with a rule of inference, which may be applied to various sentences, the temporal metaphor cannot mean 'no matter what true sentences we apply the rule to'. It must have a somewhat different meaning; Tarski's use of the phrase 'the case *when*' (K.1, K.7 Polish, italics added) suggests construing it in terms of cases, i.e. circumstances: 'whenever' would mean 'under any circumstances in which' and 'at the same time' would mean 'in the same circumstances'. Thus, the intuitively grounded condition—that it cannot happen that the implying sentences are true and at the same time the implied sentence is false—could be taken to mean that there are no circumstances in which both the implying sentences are true and the implied sentence is false. This interpretation is similar to that of Gómez-Torrente, but does not have the same implication of making the intuitively based condition of 'necessary' truth-preservation redundant. For the generality concerns the circumstances in which the implying sentences are true, not the extra-logical constants that occur in the sentences among which the relation of following obtains. On this interpretation, the intuitively based condition of necessary truth-preservation would amount to saying that, no matter what the circumstances, the implying sentences will not be true while the implied sentence is false. And the formality condition of independence of the extra-logical constants would be that this intuitive condition holds also for parallel cases obtained by uniform substitution for the extra-logical constants in the implying sentences and implied sentence.

Addressing Etchemendy's critique also requires repairing Tarski's failure, noted by several commentators, to provide in his definition for varying the domain. Absence of a counter-interpretation when the domain is fixed clearly does not amount to logical consequence: with a fixed denumerably infinite domain, for example, there is no interpretation in which '$\exists x\ x=x$' ('there is at least one object') is true but '$\exists x\ \exists y\ x\neq y$' ('there are at least two objects') is false, but 'there are at least two objects' does not follow logically from 'there is at least one object'. Tarski discussed the concept of truth in a domain in his monograph on truth (1933, 51–8, 89–90, 1935, 318–27, 31–363, 1956, 1983, 199–208, 239–41), but put this discussion in small print in the Polish and German originals, and advised his readers in a note that they could skip this discussion if they had no great interest in the special concepts and investigations from the realm of the methodology of deductive sciences. The failure to provide for variation of the domain in the present paper may thus have reflected a desire to avoid complications which would be difficult for his philosophical audience to understand.

The invariance of the domain also explains some puzzling claims in Tarski's paper.

First, Tarski claims that universal generalizations whose instances for all natural numbers are derivable from the axioms of ω-incomplete deductive theories follow logically from those axioms, even though in some cases they are not derivable from them. This discrepancy between the consequences of the syntactic conception and of his proposed model-theoretic conception is in fact his main motivation for preferring the model-theoretic conception to the syntactic one for theoretical work. But according to the contemporary model-theoretic conception of following logically, such universal generalizations do not follow logically from the axioms of the ω-incomplete theory; there are non-standard models with inaccessible cardinals in which the axioms are true but the universal generalizations false. To produce such models, however, requires varying the domain. Although, as already mentioned, Tarski recognized at the time the possibility of varying the domain of interpretation of a deductive theory, he would typically fix the domain of a deductive theory in his writings during this period by having a predicate whose extension was the set of objects that were the subject-matter of the theory, even though he recognized the possibility of reinterpreting that predicate as applying to a wholly different set of objects; see his (1936c, 1937, 1946/1941).

Second, Tarski claims (3.1) that following formally and following materially have the same extension if all terms of a language are treated as logical. As Sher (1991) points out, this claim is false on the contemporary model-theoretic conception of following logically; the pair of sentences mentioned three paragraphs previously constitute a counter-example. Tarski's claim is correct if and only if the domain is fixed. If the domain is not fixed, it is easy to generate counter-examples like Sher's. If the domain is fixed, and all terms of the language are treated as logical (i.e. not subject to replacement by corresponding variables), then the relation described in Tarski's definition obtains vacuously (as Tarski claimed) if and only if it is not the case that all the sentences of the class $\mathfrak{K}$ are true and at the same time the sentence X is false. Since no constants are replaced by corresponding variables, the sentential functions to be considered are just the sentences themselves. In such a case, we must suppose that a sequence of objects 'satisfies' a 'sentential function' (i.e. a sentence) if and only if the sentence is true; assuming a bivalent semantics, a sequence would fail to 'satisfy' a 'sentential function' if and only if the sentence is false. If all the sentences of the class $\mathfrak{K}$ are true and the sentence X is false, then every sequence of objects is a model of the class $\mathfrak{K}$ and no sequence of objects is a model of the sentence X; hence, given that there are such sequences of objects, the sentence X does not follow logically, in the sense of Tarski's definition, from the sentences of the class $\mathfrak{K}$. If not all the sentences of the class $\mathfrak{K}$ are true or the sentence X is true, or both, then no sequence of objects satisfies all the 'sentential functions' obtained from the sentences of the class $\mathfrak{K}$ but fails to satisfy the 'sentential function' obtained from the sentence X. It is of course a matter of conjecture whether Tarski had the preceding argument in mind when he asserted that following formally and following materially coincide when all constants are treated as logical.

In 2.7 Tarski claims that one can prove that his definition of following logically satisfies each of his two conditions of material adequacy expressed jointly in condition (F), and he claims that condition (F) is not sufficient for following logically as he has defined the concept. He does not provide the two proofs to which he alludes, which scholars must therefore reconstruct. His argument that

condition (*F*) is not sufficient depends on his earlier remark (2.4.4) that condition (*F*) can be satisfied in cases where a sentence does not follow formally from given sentences, simply because the language lacks names for some objects in its domain.

In 3.1 Tarski remarks that his definition leaves open a whole series of questions about the concept of following logically. He states and discusses only one of these questions, namely, the question of whether there is a completely objective basis for distinguishing logical terms from extra-logical terms. What other open questions did Tarski see? The scholarly literature on Tarski's classic paper has so far failed to address this question. The preceding paragraphs raise issues which Tarski might well have had in mind: whether to have uninterpreted extra-logical constants as well as free variables (see Hodges 1986), how to articulate the concept of a model so as to correspond to mathematicians' use of this concept, how to clarify the sense of 'must' in the intuitive condition for following logically, whether and how to incorporate variation of the domain in the concept of a model.

9. Principles of translation

Tarski notes that his (1946/1941) is a revised version of a book which appeared first in Polish and then 'in an exact German translation' (xi). We have striven to be as exact in translating Tarski from Polish (and German) to English as he was in translating himself from Polish to German. To this end, we have been guided by the following principles:

1. The translation should convey to an attentive and knowledgeable English-speaking reader familiar with the intellectual background of Tarski's paper what the Polish original conveys to a similarly attentive and knowledgeable Polish-speaking reader.

2. As in translations of ancient Greek philosophical writings into Latin, Arabic and Syriac, a reader of our translation who knows both the language of the translation and the language of the original should be able to determine on the basis of the translation alone how the original reads, particularly with respect to crucial terminology and claims. (This principle is more strict than the previous one; it rules out substantial recasting of a sentence even if the recast sentence conveys the same meaning as the original.)

3. If a given word or phrase occurs more than once in the Polish original with the same meaning, the English translation will normally use the same word or expression for it at each such occurrence, except in the case of frequently occurring particles. (A Polish-English glossary giving all such equivalences can be consulted at http://www.humanities.mcmaster.ca/~hitchckd/glossaries.htm.) For example, we translate the Polish verb *spełniać* as 'to satisfy' in all its occurrences (rather than sometimes as 'satisfy' and other times as 'fulfill', a possible alternative); but the Polish word *zakres* variously as 'denotation', 'realm' or 'scope', because its meaning varies with the context.

4. If a given word or phrase occurs more than once in the English translation with the same meaning, the Polish original will normally have the same word or expression at each corresponding occurrence, except in the case of frequently

occurring particles. (An English–Polish glossary giving all such equivalences can be consulted at http://www.humanities.mcmaster.ca/~hitchckd/glossaries.htm.) For example, we use the word 'following' rather than 'consequence' to translate the Polish *wynikanie*, in part because we decided to use 'consequence' for the Polish *konsekwencja*. (In Tarski's paper, the word *konsekwencja* appears to refer to that which follows from something, whereas the word *wynikanie* refers to the relation between that which follows and what it follows from. There is a similar distinction in the German version between the use of *Konsequenz* for that which follows from something and of *Folgerung* for the relation of following, except for a use of *Folgerung* at 2.7.3 for that which follows from something.) On the other hand, we use 'the following' to translate the Polish *następny*, *następujący*, and *dalszy* in such contexts as 'the following peculiarity', where the word 'following' clearly has a different meaning than when one speaks of 'following logically'. (We use 'the following' for three different Polish words as an exception to the normal isomorphism; it is not practical to translate each word differently, and readers of the English translation will not be misled by the many-one relation in this case.)

5. If Tarski's own writings in English, or personally approved translations into English of his works, indicate that he preferred a certain word or phrase for a given Polish word or phrase, then we use the word or phrase Tarski preferred. For example, we translate the Polish *równokształtny* and *nierównokształtny*, which literally mean 'equally-shaped' and 'non-equally-shaped' and which dictionaries render as 'isomorphic' and 'non-isomorphic' (not Tarski's meaning in this paper), by the word 'equiform', which Tarski preferred.

6. The translation will retain the punctuation of the original, except in cases where punctuation conventions of the two languages differ. Tarski's punctuation often indicates inferential relationships, e.g. in the use of a colon to introduce a supporting reason for a claim, and must be retained in such cases to convey his meaning. See for example our translation of sentences 0.1 and 0.2, in contrast to the previous English translation.

7. Where feasible, words with common roots will be translated by words with common roots, as can be seen by inspecting successive entries in the Polish–English glossary (available at http://www.humanities.mcmaster.ca/~hitchckd/glossaries.htm) which have the same root. This principle is our main reason for the unusual translation of *wynikanie logiczne* as 'following logically' rather than as 'logical consequence'. Our translation preserves the common root with the verb *wynikać*, 'to follow', which Tarski uses in his definition. Our translation is not quite literal; literally, *wynikanie logiczne* would be translated as 'logical following' (and *wynikanie materialne* as 'material following' and *wynikanie formalne* as 'formal following'), but this translation is not standard English. We thank John Corcoran for the suggestion of transforming Tarski's adjectives into adverbs to get closer to standard English usage. In trying to preserve commonality of roots in our translation where it exists in the Polish, we are following Tarski's own

practice with respect to the Polish and German versions of his paper; almost always, if two Polish words in his paper have a common root, so will the two corresponding German words, and vice versa.

8. We have used each version to disambiguate the other. For example, at 3.3.1 we translate the German *wichtige* as 'weighty' rather than 'important', because the Polish *ważkich* can only mean the former; in the context, in fact, 'weighty' makes more sense than 'important'. Definite and indefinite articles in the German have provided a guide in disambiguating the Polish, which has no articles. A particularly challenging instance was Tarski's use in German in the parenthetical clause in 2.5.9 of 'vom Modell', which is an apparent contraction of 'von dem Modell', literally 'of the model'. The implication that a system of axioms of a deductive theory has exactly one model was clearly known by Tarski to be false; it is a key point of his introduction to logic (1936c, 1937, 1946/1941) that a system of axioms can have more than one model, and Tarski's work in semantics is the foundation of contemporary work in model theory. Informally, however, 'vom' can be used as a contraction for 'von einem', 'of a', and we were able to find places in Tarski's German writings of the 1930s where he used 'vom' in contexts where it could only be translated as 'of a' (1933b, 97, 1937, 94). So that is how we translated it at 2.5.9.

9. Although we have made no special effort to harmonize the vocabulary of our translation with that of the previous English translation, we did cross-check the penultimate version of our translation against the previous translation as a way of making sure that there were no errors in our translation of the German version.

The translation is a joint effort.

10. Tarski's language

The process of translating a text written in two languages by the same person into a third language offers a rare insight into how the same ideas and concepts may find their linguistic representation in various languages. Tarski was able to exploit the means available in Polish and German—despite their differences—to represent in an almost identical way what he intended to express. It is difficult to judge today to what extent he made a conscious effort to find in both languages words and phrases that would allow for or give rise to the same associations and imagery, or whether the almost perfect equivalence of the two texts was simply a result of constructing a scientific argument in two languages at the same time. The result is truly astounding. It has been a treat for a linguist and German philologist, whose native language is Polish, to witness and experience this dimension of Tarski's writing. The translators tried very hard to emulate this effect in their English version but are aware that they were not always able to find a fully satisfactory solution.

One reason why this was not always feasible is the difference between the traditions of scientific discourse in Polish and German on the one hand, and the traditions in English on the other hand. In the 1930s, both Polish and German scientists and researchers were striving for scientific terminology that would be

native to their respective languages. Polish had never been subjected to the same extent as German to linguistic purism. But, perhaps because of Poland's newly regained national independence, perhaps because some of their research was at the foundation of their field, Polish scientists made an effort to keep their scientific vocabulary as close to everyday language and as Polish as possible. Thus, both in Polish and in German, Tarski uses a lexicon that would be understood by both specialists and non-specialists, with hardly any foreign scientific terms. For example, the Polish word *poprzednik*, which we had to translate as 'antecedent', means simply 'predecessor' or 'the one before', and could be used to refer to e.g. the person who had the same position before the one who has it now. Similarly, the corresponding German word *Vorglied* consists of the root 'element' or 'member' and a prefix corresponding to the English 'before'. English in contrast, because of its different historical development, tends to use high-faluting words for scientific concepts. Thus, while in both Tarski's texts the thesis about everyday language as a test finds its reflection in the texts themselves, in English we talk about everyday language but use scientific terminology that would be obscure for people from outside the discipline.

It may also be of interest to note that, while both Polish and German scientific discourse gives preference to an impersonal presentation, there are differences in Tarski's systems of self-reference that cannot be explained solely by a difference in the grammatical systems of the two languages. It would be customary for scientists writing in either Polish or German in the 1930s to use the first person plural as a means of self-reference, as well as to use more indirect means of self-reference, such as passive constructions and impersonal reflexive constructions. The two systems are almost parallel in their preferences. On a number of occasions, however, Tarski chooses an impersonal construction in German where, in Polish, he uses the first person plural. It is possible that he wanted to make sure in the German version that the inclusiveness of the first person plural construction was not misread by anyone as suggesting that the author assumes that the reader (or hearer at the conference) would share his perspective. It may also be worth stressing that Tarski's fairly frequent direct self-reference (i.e. use of the first person singular pronoun *I*) was not at all typical for that time. We have tried to capture these differences and point to them in the footnotes.

We hope that our translation will allow the English-speaking reader to experience Tarski's high appreciation for the expressive force of language and his meticulous approach to the choice of every single word.

Acknowledgements

Hitchcock presented previous versions of this introduction at the University of Calgary, State University of New York at Buffalo and McMaster University. Thanks are due to comments received there, and in correspondence, from John Corcoran, Brian Chellas, Ali Kazmi, Jan Woleński, Jacek Pasniczek, Wolfgang Fischer, Greg Moore, Jeffery Zucker, Michael Scanlan and Bernhard Banaschewski.

On the Concept of Following Logically

ALFRED TARSKI

[0. Introduction]

[0.1] The concept of f o l l o w i n g l o g i c a l l y belongs to the category of those concepts[1] whose introduction into the domain of exact[2] formal investigations was not only an act[3] of arbitrary decision on the side of this or that researcher: in making precise the content of this concept, efforts were made[4] to conform to the everyday 'pre-existing' way it is used.[5] **[0.2]** This task was accompanied by the difficulties usual[6] in such situations:[7] the concept[8] of following is not distinguished[9] from other concepts of everyday language by a clearer content or more precisely delimited denotation, the way it is used is unstable,[10] the task of capturing and reconciling all the[11] murky, sometimes contradictory intuitions[12] connected with that concept[13] has to be acknowledged a priori as unrealizable,[14] and one has to reconcile oneself in advance to the fact that every precise definition of the concept under consideration will to a greater or lesser degree bear the mark of arbitrariness.[15]

[1. The syntactic approach]

[1.1 Its initial success] [1.1.1] Even relatively[16] recently it seemed to many logicians[17] that they had managed, with the help of a relatively simple conceptual apparatus,[18] to capture almost precisely the everyday[19] content of the concept of following, or rather to define a new concept which with respect to its denotation would coincide[20] with the everyday concept. **[1.1.2[** This belief arose[21] on the basis

1 *the category of those concepts*/German: *that category of concepts*
2 *exact*/German: *rigorous*
3 *only an act*/German: *a matter*
4 *in making precise the content of this concept, efforts were made*/German: *in making this concept precise, one has tried hard*
5 *the everyday 'pre-existing' way it is used*/German: *its common way of being used, already found in everyday language*
6 *usual*/German: *that usually occur*
7 *situations*/German: *cases*
8 *the concept*/German: *the common concept*
9 *not distinguished*/German: *not positively distinguished*
10 *by a clearer content or more precisely delimited denotation, the way it is used is unstable,*/German: *by the clarity of its content, its denotation is not sharply delimited and its usage in language is unstable*
11 *the task of capturing and reconciling all the*/German: *an attempt to reconcile all possible*
12 *intuitions*/German: *tendencies*
13 *that concept*/German: *the use of this concept*
14 *has to be acknowledged a priori as unrealizable*/German: *is definitely unfeasible*
15 *bear the mark of arbitrariness*/German: *exhibit arbitrary features*
16 *relatively*/Absent in German
17 *it seemed to many logicians*/German: *many logicians believed*
18 *simple conceptual apparatus*/German: *small expenditure of concepts*
19 *everyday*/German: *common*
20 *would coincide*/German: *coincides*
21 *This belief arose*/German: *Such a belief could develop namely*

of the newer achievements of the methodology of the deductive sciences.[22] **[1.1.3]** Thanks to the development[23] of mathematical logic, we have learned during recent decades to present mathematical sciences in the form of formalized deductive theories. **[1.1.4]** In these theories, as is well known, the proof of each theorem reduces to single or multiple application of a few simple rules of inference—such as the rule of substitution or detachment—rules which instruct us to which operations[24] of a purely structural character (i.e. operations[25] involving exclusively the external structure of the sentences[26]) one has to subject axioms of the theory or previously proven theorems[27] in order that the sentences obtained as a result of those operations[28] may also[29] be acknowledged as proven. **[1.1.5]** Logicians began to suppose[30] that those[31] few rules of inference completely exhaust[32] the content of the concept of following: whenever a sentence follows from others, it can[33] be obtained from them—by a more or less complicated route—[34]with the help of the operations[35] specified in these[36] rules. **[1.1.6]** In defence of their position[37] against sceptics who expressed doubt whether[38] the concept of following formalized in this way really[39] coincides with respect to its denotation with the everyday concept,[40] logicians could put forward one weighty argument: namely, they[41] in fact succeeded in presenting all exact reasonings carried out from time immemorial in the field of mathematics in the form of formalized proofs completely contained within the framework of the constructed deductive theories.[42]

[1.2 ω-incomplete deductive theories] [1.2.1] Nevertheless, today we are already[43] aware that the scepticism was here not at all out of place[44] and that the position sketched above[45] cannot be maintained. **[1.2.2]** Already a few[46] years ago, I gave an

22 *the deductive sciences*/German: *deductive science*
23 *development*/German: *advances*
24 *operations*/German: *transformations*
25 *operations*/German: *transformations*
26 *involving exclusively the external structure of the sentences*/German: *in which exclusively the external structure of the sentences is considered*
27 *axioms of the theory or previously proven theorems*/German: *axioms or already proven theorems of the theory*
28 *operations*/German: *transformations*
29 *also*/German: *themselves*
30 *began to suppose*/German: *now thought* ('Now' translates *nur*, 'only', an apparent misprint for *nun*, 'now')
31 *those*/German: *these*
32 *completely exhaust*/ German: *exhaust*
33 *can*/German: *could*
34 *—by a more or less complicated route—*/German: *—so one thought—by a more or less complicated route*
35 *operations*/German: *transformations*
36 *these*/German: *the*
37 *In defence of their position*/German: *In order to defend this position*
38 *expressed doubt whether*/German: *doubted that*
39 *really*/Absent in German
40 *the everyday concept*/German: *the common one*
41 *namely, they*/German: *the circumstance that one*
42 *completely contained within the framework of the constructed deductive theories*/ Absent in German
43 *already*/Absent in German
44 *here not at all out of place*/German: *very much in place*
45 *above*/Absent in German
46 *a few*/German: *several*

example—[47]by the way a quite elementary one—[48]of a deductive theory[49] which exhibits the following peculiarity: among the axioms or theorems[50] of this theory there occur sentences of the form:[51]

A_0. *0 possesses the property*[52] P[53]
A_1. *1 possesses the property*[54] P
A_2. *2 possesses the property* P[55]

and so on, more generally[56] all particular sentences of the form:

A_n. *n possesses the property*[57] P

where '*n*' stands for an arbitrary symbol designating a natural number in some specified (e.g. decimal) system of numbering, but despite this[58] the universal sentence:

A. *every natural number possesses the property*[59] P[60]

cannot be proven on the basis of the theory under consideration with the help of the rules of inference normally used.[61,A] **[1.2.3]** This fact attests by itself, I think, that[62] the formalized concept of following, which until now[63] was generally used in the construction of deductive theories,[64] by no means coincides with the everyday concept—after[65] all, from the point of view of everyday intuitions[66] it seems indubitable[67] that the sentence[68] *A* follows[69] from the totality of sentences[70] A_0, A_1, ... A_n, ... : whenever[71] all these sentences are true, then also the sentence *A* must be true.

47 —/German,
48 —/German,
49 *a deductive theory*/German: *a theory of that kind*
50 *axioms or theorems*/German: *theorems*
51 *sentences of the form*/German: *such sentences as*
52 *property*/German: *given property*
53 Tarski uses *W* in Polish (abbreviating *własność*, 'property') and *E* in German (abbreviating *Eigenschaft*, 'property')
54 *property*/German: *given property*
55 A_2. *2 possesses the property P*/Absent in German
56 *more generally*/German: *in general*
57 *property*/German: *given property*
58 *, but despite this*/German: *; on the other hand*
59 *59 property*/German: *given property*
60 *P*/German: *P,*
61 *rules of inference normally used*/German: *normal rules of inference* (superscripted capital letters, such as [A] above, refer to Tarski's own notes, printed at the end of this translation)
62 *attests by itself, I think, that*/German: *speaks, as it seems to me, for itself: it shows that*
63 *which until now*/German: *as it*
64 *in the construction of deductive theories*/German: *by mathematical logicians*
65 *the everyday concept—after*/German: *the common one. After*
66 *from the point of view of everyday intuitions*/German: *intuitively*
67 *indubitable*/German: *to be certain*
68 *sentence*/German: *universal sentence*
69 *follows*/German: *follows in the common sense*
70 *sentences*/German: *all particular sentences*
71 *: whenever*/German: *; if only*

[1.3 Extensions to accommodate ω-incomplete theories] [1.3.1] It then turned out[72] to be possible to formulate new rules of inference, rules[73] which do not differ in their logical character[74] from the old[75] ones, which intuitively are as[76] infallible as the old ones,[77] i.e. always lead from true sentences to other[78] true sentences, but which cannot be reduced to the old[79] rules, because they make possible the proof of sentences which it was impossible to prove with the help of the old rules.[80] **[1.3.2]** One of the rules of such a character[81] is closely connected with the example described above; it[82] is the so-called rule of infinite induction, which states that, whenever all sentences A_0, A_1, ... A_n, ... are proven, then one may also acknowledge as proven the sentence A[83] (the symbols 'A_0', 'A_1' and so on[84] we use[85] here in the same[86] sense as before).[A] **[1.3.3]** This rule still differs essentially from the old rules with respect to[87] its 'infinitistic' character: one can use it in a theory only[88] when one has already succeeded in proving infinitely many sentences of this theory, and such a situation after all can never occur in practice.[89] **[1.3.4]** It is not difficult however,[90] by a certain transformation of the rule under consideration, to remove this shortcoming; for this purpose we take into consideration sentence B, which states that all the sentences A_0, A_1, ... A_n, ... are provable (and not that they in fact have been proven) with the help of the hitherto existing rules of inference,[91] and we formulate[92] the following rule[93] (which however is not precisely equivalent to the former rule):[94] whenever[95] the sentence B is proven, then one may[96] acknowledge as proven the corresponding sentence A. **[1.3.5]** Here we can[97] still be met with the

72 *It then turned out*/German: *In connection with the fact just described, it turned out*
73 *rules*/Absent in German
74 *character*/German: *structure*
75 *old*/German: *older*
76 *as*/German: *just as*
77 *as the old ones*/Absent in German
78 *other*/German: *new*
79 *old*/German: *older*
80 *because they make possible the proof of sentences which it was impossible to prove with the help of the old rules*/Absent in German
81 *of the rules of such a character*/German: *example of such rules*
82 *is closely connected with the example described above; it*/Absent in German
83 *which states that, whenever all sentences A_0, A_1, ... A_n, ... are proven, then one may also acknowledge as proven the sentence A*/German: *according to which one may consider the sentence A as proven, if only all sentences A_0, A_1, ... A_n, ... were proven*
84 The German has 'w.s.w.', a misprint for 'u.s.w.'
85 *we use*/German: *are used*
86 The German has 'im demselben', a misprint for 'in demselben'
87 *This rule still differs essentially from the old rules with respect to*/German: *This rule of inference is however still distinguished essentially from the older rules by*
88 *one can use it in a theory only*/German: *it can only be used in the construction of a theory*
89 *and such a situation after all can never occur in practice.*/German: *—a situation which after all never occurs in practice.*
90 *not difficult however*/German: *however easy*
91 *are provable (and not that they in fact have been proven) with the help of the hitherto existing rules of inference*/German: *are* provable *on the basis of the hitherto existing rules of inference (and not that they in fact have been proven)*
92 *formulate*/German: *then put forward*
93 *rule*/German: *rules*
94 *(which however is not precisely equivalent to the former rule)*/Absent in German
95 *whenever*/German: *if only*
96 *may*/German: *may also*
97 *can*/German: *could*

reproach that the sentence *B* is in no way one of the sentences[98] of the theory which we are constructing,[90] that[100] it belongs to the realm of[101] the so-called metatheory (i.e. of the science whose object of investigation is the given theory[102]), that therefore[103] the application in practice of the above rule[104] always requires a transition[105] from the theory to the metatheory.[B] **[1.3.6]** In order to avoid this reproach, we limit ourselves to only[106] those deductive theories in which the arithmetic of the natural numbers can be constructed,[107] and we direct our attention to the fact that in each such theory all the concepts[108] of the corresponding metatheory can be interpreted (and this because of the possibility of setting up[109] a one-to-one correspondence between the expressions of a language and the natural numbers).[B] **[1.3.7]** We can thus replace in the rule under consideration the sentence *B* by the sentence *C*,[110] which is the interpretation of sentence *B* on the basis of arithmetic; in this way we come to a rule which does not differ essentially, either with respect to the conditions of its applicability, or with respect to the character of the concepts occurring in its formulation, or finally with respect to the degree of[111] its intuitive infallibility, from the rules used until now,[112] it is only more complicated than they are.[113] **[1.3.8]** What is more, one can provide arbitrarily many such rules.[114] **[1.3.9]** It is sufficient, as a matter of fact, just[115] to direct one's attention to the fact that the rule formulated above was[116] essentially dependent on the denotation of the concept 'sentence provable with the help of[117] the hitherto existing rules'; in accepting[118] this rule, we are thereby widening[119] the denotation of the concept indicated,[120] for the widened denotation we[121] can thus construct[122] a new analogous rule, and so on without end.[123] **[1.3.10]** It would be interesting to investigate whether the rules[124] that were used until now occupy,[125] for these or

98 *one of the sentences*/German: *a sentence*
99 *we are constructing*/German: *is being constructed*
100 *that*/German: *but that*
101 *the realm of*/Absent in German
102 *of the science whose object of investigation is the given theory*/German: *to the study of the theory under consideration*
103 *therefore*/German: *consequently*
104 *above rule*/German: *rule in question*
105 The German has 'Ubergang', a misprint for 'Übergang'.
106 *we limit ourselves to only*/German: *we want to limit ourselves only to*
107 *constructed*/German: *established*
108 *concepts*/German: *concepts and sentences*
109 *and this because of the possibility of setting up*/German: *since one can set up*
110 *C*/German: *B'*
111 *the degree of*/Absent in German
112 *the rules used until now,*/German: *the hitherto common rules*
113 *it is only more complicated than they are*/German: *(although it is considerably more complicated)*
114 *What is more, one can provide arbitrarily many such rules*/German: ¶*It is now possible to provide still other rules of like character and indeed arbitrarily many*
115 *just*/Absent in German
116 *was*/German: *is*
117 *with the help of*/German: *on the basis of*
118 *in accepting*/German: *if one accepts*
119 *we are thereby widening*/German: *one thereby widens*
120 *the concept indicated*/German: *the aforesaid concept*
121 *we*/German: *one*
122 *construct*/German: *put forward*
123 *without end*/German: *ad infinitum*
124 *the rules*/German: *to the rules*
125 *occupy*/German: *there is to be attributed*

other objective reasons, a distinguished privileged[126] position among all possible rules of inference.[127]

[1.4 Failure: Gödel's incompleteness theorem] [1.4.1] The supposition suggests itself[128] that on the route sketched above—supplementing the rules of inference used in the construction of deductive theories with further rules of a structural character[129]—we[130] would succeed finally in capturing[131] the 'essential' content[132] of the concept of following, which has by no means been exhausted by the rules used until now.[133] **[1.4.2]** Relying on the investigations of K. Gödel,[C] one can demonstrate that this supposition is mistaken: if we abstract from certain theories with a very elementary structure,[134] then always—[135]no matter how we enrich the stock of rules of inference—[136]we shall be able to construct[137] sentences which follow in the everyday[138] sense from the theorems of the deductive theory under consideration, but which cannot be proven in this theory on the basis of the accepted rules.[D] **[1.4.3]** In order to obtain the proper concept of following, essentially close to the everyday[139] concept, one must resort in its definition to other methods altogether[140] and[141] use a quite distinct[142] conceptual apparatus.[143] **[1.4.4]** It is perhaps not superfluous to remark in advance that—in comparison to the proper concept of following—the old one,[144] generally used until now by mathematical logicians, by no means loses its importance: this concept, possibly widened with the help of new rules of inference,[145] will probably always play a decisive role in practice, in the construction of deductive theories,[146] as an instrument which allows one to prove or refute individual sentences of the theories being constructed;[147] it seems on the other hand that one should put the proper concept of following in the foreground[148] in considerations of a general theoretical character.[E]

126 *privileged*/Absent in German
127 *among all possible rules of inference*/Absent in German
128 *suggests itself*/German: *now suggests itself*
129 *with further rules of a structural character*/Absent in German
130 *we*/German: *one*
131 *finally in capturing*/German: *in fully grasping*
132 *"essential" content*/German: *'content'*
133 *which has by no means been exhausted by the rules used until now*/Absent in German
134 *with a very elementary structure*/German: *of a particularly elementary character*
135 *always*—/Absent in German
136 *we enrich the stock of rules of inference*—/German: *the hitherto existing rules of inference are supplemented by new purely structural rules,*
137 *we shall be able to construct*/German: *it is possible to construct in each deductive theory*
138 *everyday*/German: *common*
139 *everyday*/German: *common*
140 *other methods altogether*/German: *quite different methods*
141 *and*/Both the Polish and the German have just a comma here
142 *distinct*/German: *different*
143 The German version, but not the Polish, has a long dash between these two sentences
144 *the proper concept of following—the old one*/German: *the new one—the old concept of following*
145 *, possibly widened with the help of new rules of inference,*/Absent in German
146 *play a decisive role in practice, in the construction of deductive theories,*/German: *retain a decisive significance for the practical construction of deductive theories—*
147 *the theories being constructed*/German: *these theories*
148 *one should put the proper concept of following in the foreground*/German: *the proper concept of following is to be put in the foreground*

[2. The semantic approach]

[2.1 Carnap's definition] [2.1.1] The first attempt at the formulation of a precise definition for[149] the proper concept of following comes from R. Carnap;[150] this attempt however is quite essentially tied to the specific properties of the formalized language which was selected as object[151] of the investigation. **[2.1.2]** The definition proposed by Carnap can be formulated[152] in the following way:

> *We say that*[153] *the sentence X follows logically from the class of sentences* $\mathfrak{K}$[154] *if and only if the class consisting*[155] *of all sentences of the class* $\mathfrak{K}$ *and of the negation of the sentence X is contradictory.*

[2.1.3] Clearly, the whole weight of the above definition rests on the concept of contradictoriness (contradiction);[156] the definition of this concept provided by Carnap bears[157] however too special and complicated[158] a character for it to be possible to cite it here without long and rather burdensome[159] preparatory considerations.[F]

[2.2 An alternative based on scientific semantics] [2.2.1] I would like to sketch here a general method which makes it possible, as it seems to me, to construct a formally correct and materially adequate[160] definition of the concept of following for an extensive category[161] of formalized languages. **[2.2.2]** I would like to stress[162] that the conception[163] of following which I intend to develop[164] makes no exaggerated claims[165] to complete originality: the intuitions[166] inherent in it will undoubtedly[167] be sensed by many a logician who considered the concept of following logically and attempted[168] to characterize it more closely as something well-known or even as something of his own. **[2.2.3]** I have the impression[169] however that only the methods developed in recent years in establishing scientific semantics, and the concepts which one succeeded in making precise[170] with the help of these

149 *for*/German: *of*
150 The German version inserts note [F] at this point, as well as at the point two sentences later where the Polish version inserts it
151 The German version misprints 'Gegenständ' instead of 'Gegenstand'
152 *formulated*/German: *rendered*
153 *We say that*/Absent in German
154 *the class of sentences* $\mathfrak{K}$/German: *the sentences of the class K*
155 *consisting*/German: which consists
156 *Clearly, the whole weight of the above definition rests on the concept of contradictoriness (contradiction)*/ German: *The crucial point of the definition just formulated clearly lies in the concept 'contradictory'*
157 *bears*/German: *exhibits*
158 *special and complicated*/German: *complicated and special*
159 *burdensome*/German: *troublesome*
160 *a formally correct and materially adequate*/German: *an adequate*
161 *category*/German: *class*
162 *I would like to stress*/German: *At the same time, I want to stress*
163 *conception*/German: *conception of the concept*
164 *which I intend to develop*/German: *to be developed here*
165 *claims*/German: *claim*
166 *intuitions*/German: *tendencies*
167 *undoubtedly*/German: *certainly*
168 *considered the concept of following logically and attempted*/German: *has attempted to subject the concept of following logically to a more precise investigation and*
169 *I have the impression*/German: *It seems to me*
170 *which one succeeded in making precise*/German: *made precise*

methods,[171] make it possible[172] to put those intuitions[173] into a form which is exact and not subject to reproach.[174.G]

[2.3 A two-part necessary condition of material adequacy] **[2.3.1]** The point of departure for us will be certain considerations of an intuitive nature. **[2.3.2]** Let us consider an arbitrary class of sentences $\mathfrak{K}$[175] and an arbitrary[176] sentence X which follows from the sentences of this class. **[2.3.3]** From the point of view of everyday intuitions[177] it is clear that it cannot happen[178] that all the sentences of the class $\mathfrak{K}$ would be true[179] but at the same time the sentence X would be[180] false. **[2.3.4]** Since moreover it is a question here of the relation[181] of following logically, i.e. formally, and therefore of a relation which has to be completely[182] determined by the form of the sentences among which it obtains, thus following[183] cannot depend on[184] our knowledge of the external world,[185] in particular on our knowledge[186] of the objects which are spoken about in the sentences of the class $\mathfrak{K}$[187] or in the sentence X,[188] cannot be lost[189] as a result of our[190] replacing the names[191] of these objects[192] in the sentences under consideration by names[193] of other[194] objects. **[2.3.5]** Both these[195] circumstances, which seem highly[196] characteristic and essential for the concept[197] of following, find jointly their expression in the following condition:[198]

(F)[199] *If in the sentences of the class* $\mathfrak{K}$[200] *and in the sentence* X *we replace*[201] *the constant terms*[202] *which are not general-logical terms correspondingly*[203] *by*

171 *the help of these methods,*/German: *their help*
172 *make it possible*/German: *allow one*
173 *those intuitions*/German: *these tendencies*
174 *a form which is exact and not subject to reproach*/German: *an exact form*
175 $\mathfrak{K}$/German: K
176 *an arbitrary*/German: *a*
177 *From the point of view of everyday intuitions*/German: *From an intuitive standpoint*
178 *it is clear that it cannot happen*/German: *it can never happen*
179 *all the sentences of the class* $\mathfrak{K}$ *would be true*/German: *the class K consists of nothing but true sentences*
180 *would be*/German: *is*
181 *relation*/German: *concept*
182 *completely*/German: *uniquely*
183 *following*/German: *this relation*
184 *cannot depend on*/German: *can in no way be influenced by*
185 *our knowledge of the external world,*/German: *empirical knowledge and*
186 *on our knowledge*/German: *by the knowledge*
187 $\mathfrak{K}$/German: K
188 *,*/German: *:*
189 *cannot be lost*/German: *the relation of following cannot be destroyed*
190 *our*/German: *one*
191 *names*/German: *designations*
192 *these objects*/German: *the objects mentioned*
193 *by names*/German: *everywhere by designations*
194 *other*/German: *some other*
195 *these*/German: *the cited*
196 *highly*/German: *to be very*
197 *concept*/German: *proper concept*
198 *find jointly their expression in the following condition*/German: *we can combine in the sentence*
199 Tarski uses W in Polish (for *wynikanie*) and F in German (for *Folgerung*)
200 $\mathfrak{K}$/German: K
201 *we replace*/German: *one replaces*
202 *constant terms*/German: *constants*
203 *which are not general-logical terms correspondingly*/German: *—with the exception of the purely logical ones—*

arbitrary other constant terms[204] *(where we replace equiform constants everywhere by equiform constants*[205]*) and in this way we obtain a new class of sentences* $\mathfrak{K}'$ *and a new sentence* X'[206]*, then the sentence* X' *must be true if only all sentences of the class* $\mathfrak{K}'$[207] *are true.*

[2.3.6] [For the purpose of simplifying our considerations,[208] we abstract—here and in what follows—from certain complications of a secondary, rather technical nature,[209] connected on the one hand with the theory of logical types and on the other hand with the necessity of the prior elimination of defined terms.[210]]

[2.4 Insufficiency of this necessary condition] [2.4.1] In this way[211] we have obtained a necessary condition for the sentence X to follow from the sentences of the class $\mathfrak{K}$; the[212] question arises[213] whether this condition is also sufficient. **[2.4.2]** If it were so,[214] the problem of the construction of[215] an adequate definition for[216] the concept of following would[217] be positively decided; only the term "*true*" occurring in the condition (F) could still cause a difficulty,[218] it is well known however that this term can[219] be defined correctly and adequately[220] on the basis of semantics.[G]

[2.4.3] Unfortunately, the matter does not seem so simple.[221] **[2.4.4]** It is not difficult to show[222] by concrete examples of formalized languages that despite satisfaction of condition (F)[223] the sentence X by no means must follow—from the point of view of everyday intuitions—[224] from the sentences of the class $\mathfrak{K}$:[225] this condition can in fact be satisfied only as a result of an insufficient stock of constant terms in the language which our considerations concern.[226] **[2.4.5]** One could only acknowledge[227] the condition (F) as sufficient for the sentence X to follow

204 *constant terms*/German: *constants*
205 *we replace equiform constants everywhere by equiform constants*/German: *everywhere like signs are replaced by like*
206 *in this way we obtain a new class of sentences* $\mathfrak{K}'$ *and a new sentence* X'/German: *one designates the class of sentences obtained thereby from K by "K'" and the sentence obtained from X by "X'"*
207 $\mathfrak{K}'$/German: *K'*
208 *our considerations*/German: *the consideration*
209 *complications of a secondary, rather technical nature*/German: *incidental complications*
210 *the prior elimination of defined terms*/German: *eliminating, i.e. replacing by undefined ones, the defined signs possibly occurring in the sentences under consideration*
211 *In this way*/German: *In the sentence (F)*
212 *the sentences of the class* $\mathfrak{K}$*; the*/German: *the class K of sentences. The*
213 *arises*/German: *now arises*
214 *it were so*/German: *this question were to be answered affirmatively*
215 *the construction of*/German: *providing*
216 *for*/German: *of*
217 *would*/German: *would thereby*
218 *decided; only the term 'true' occurring in the condition (F) could still cause a difficulty.*/German: *decided. The only difficulty would still be connected with the term 'true' which occurs in the condition (F)*
219 *it is well known however that this term can*/German: *this term can however*
220 *correctly and adequately*/German: *exactly and materially correctly*
221 *does not seem so simple*/German: *is however not so favourable*
222 *It is not difficult to show*/German: *It can and will namely occur—it is not difficult to show this*
223 *that despite satisfaction of condition (F)*/German: *—that*
224 *by no means must follow—from the point of view of everyday intuitions—*/German: *does not follow in the common sense*
225 $\mathfrak{K}$/German: *K, although condition (F) is satisfied*
226 *as a result of an insufficient stock of constant terms in the language which our considerations concern*/German: *because the language which the consideration concerns does not have a sufficient stock of extra-logical constants*
227 *acknowledge*/German: *regard*

formally[228] from the sentences of the class $\mathfrak{K}$[229] if one assumed that the language under consideration contains in itself the names[230] of all possible objects; this assumption is however fictitious, it[231] can never be realized.[H] **[2.4.6]** We must[232] look for a means of rendering[233] the intentions inherent in condition (*F*), a means which would not be dependent on[234] that fictitious assumption.[235]

[2.5 Preliminary definitions] [2.5.1] Semantics delivers such a means to us. **[2.5.2]** One of the most important concepts of semantics is[236] the concept of the satisfaction of a sentential function by individual objects or—more exactly—by a sequence of objects.[237] **[2.5.3]** It would be superfluous to explain here more closely the content of that concept:[238] the sense[239] of such turns of phrase as "[240]*John and Peter satisfy jointly*[241] *the condition*: '[242]*X and Y are brothers*'"[243] or [244] "[245]*the triple of numbers 2, 3 and 5 satisfies the equation* '[246]$x+y=z$'"[247] surely arouses doubts in no one.[248] **[2.5.4]** The concept of satisfaction—like other semantic concepts—must always be relativized to a specified language; its precise definition depends in its particulars on the structure of the language to which this concept refers.[249] **[2.5.5]** There exists however[250] a general method which makes possible the construction of such definitions for an extensive category of formalized languages; unfortunately, it would be impossible[251] to sketch here the method mentioned even in its most general features.[252,G]

[2.5.6] One of the concepts which can be defined with the help of the concept of satisfaction is the concept of model. **[2.5.7]** Let us assume that, in the language which we are considering, to each extra-logical constant correspond certain variable symbols,[253] and this in such a way that, by replacing in an arbitrary sentence a

228 *formally*/Absent in German
229 *the sentences of the class* $\mathfrak{K}$/German: *the class of sentences K*
230 *one assumed that the language under consideration contains in itself the names*/German: *in the language under consideration occurred the designations*
231 *, it*/German: *and*
232 *must*/German: *must thus*
233 *of rendering*/German: *which allows us to render*
234 *, a means which would not be dependent on*/German: *and which at the same time is completely independent of*
235 *assumption*/German: *supposition*
236 *One of the most important concepts of semantics is*/German: *To the fundamental concepts of semantics belongs*
237 *—more exactly—by a sequence of objects*/German: *rather by a sequence of objects*
238 *:*/German: *;*
239 *the sense*/German: *the intuitive sense*
240 "/Absent in German
241 *jointly*/Absent in German
242 '/German: "
243 '"/German: "
244 *or*/German: ,
245 "/Absent in German
246 '/German: "
247 '"/German: "/
248 *surely arouses doubts in no one*/German: *can however arouse no doubt*
249 *the language to which this concept refers*/German: *the given language*
250 *There exists however*/German: *One can however provide*
251 *impossible*/German: *impossible on technical grounds*
252 *the method mentioned even in its most general features*/German: *—be it only in general terms—the method mentioned*
253 *variable symbols*/German: *variables*

constant by a corresponding variable, we transform this sentence into a sentential function.[254] **[2.5.8]** Let us further consider[255] an arbitrary class of sentences $\mathfrak{L}$[256] and let us replace[257] all extra-logical constants occurring in the sentences of the class $\mathfrak{L}$[258] by corresponding variables (equiform constants by equiform variables, non-equiform by non-equiform);[259] we shall obtain[260] a class of sentential functions $\mathfrak{L}'$.[261] **[2.5.9]** An arbitrary sequence of objects which satisfies each sentential function of the class $\mathfrak{L}'$[262] we shall call a model of the class $\mathfrak{L}$[263] (in just this sense one usually speaks about a model of the system of axioms of a deductive theory); if[264] in particular the class $\mathfrak{L}$[265] consists of only one[266] sentence X, we will simply speak about a model of the sentence X.[267]

[2.6 Definition of following logically] [2.6.1] Using the concept of a model, we formulate the following definition of following logically:[268]

We say that[269] *the sentence X follows logically from the sentences of the class* $\mathfrak{K}$[270] *if and only if every model of the class* $\mathfrak{K}$[271] *is at the same time a model of the sentence X.*[I]

[2.7 Material adequacy of the definition] [2.7.1] I have the impression that everyone[272] who understands the content of the above definition[273] will[274] admit that it captures many intuitions manifested in the everyday usage of the concept of following.[275] **[2.7.2]** Its various consequences speak no less strongly for the adequacy of this definition.[276] **[2.7.3]** In particular e.g. one can[277] prove on the basis of the definition accepted above[278] that a sentence which follows logically

254 *, by replacing in an arbitrary sentence a constant by a corresponding variable, we transform this sentence into a sentential function*/German: *an arbitrary sentence becomes a sentential function if in it constants are replaced by corresponding variables*
255 *Let us further consider*/German: *Let L now be*
256 $\mathfrak{L}$/Absent in German
257 *and let us replace*/German: *. We replace*
258 $\mathfrak{L}$/German: *L*
259 *variables (equiform constants by equiform variables, non-equiform by non-equiform)*/German: *variables, that is equal constants by equal variables, different by different*
260 *we shall obtain*/German: *thereby we arrive at*
261 $\mathfrak{L}'$/German: *L'*
262 $\mathfrak{L}'$/German: *L'*
263 *model of the class* $\mathfrak{L}$/German: *model or realization of the class of sentences L*
264 *theory); if*/German: *theory). If*
265 $\mathfrak{L}$/German: *L*
266 *only one*/German: *the single*
267 *we will simply speak about a model of the sentence X*/German: *we will call the model of the class L also a model of the sentence X*/In Tarski (1983), Tarski changed the translation in Tarski (1956), *we shall also call the model of the class L the model of the sentence X*, to the following: *we shall also refer to a model of the class L as a model of the sentence X*
268 *Using the concept of a model, we formulate the following definition of following logically*/German: *On the basis of this conceptual construction, the concept of following logically can be defined in the following way*
269 *We say that*/Absent in German
270 $\mathfrak{K}$: German: *K*
271 $\mathfrak{K}$: German: *K*
272 *I have the impression that everyone*/German: *As it seems to me, someone*
273 *the above definition*/German: *the definition just cited*
274 *will*/German: *must*
275 *captures many intuitions manifested in the everyday usage of the concept of following.*/German: *fits quite well with common linguistic usage;*
276 *Its various consequences speak no less strongly for the adequacy of this definition*/German: *this comes to light to an even stronger degree from its various consequences*
277 *In particular e.g. one can*/German: *So one can in particular*
278 *the definition accepted above*/German: *this definition*

from[279] true sentences must itself[280] be true; further, that the relation of following logically[281] is completely independent of the sense of the extra-logical constants occurring in the sentences among which this relation obtains;[282] in a word, one[283] can show that the above formulated condition (*F*) is necessary for the sentence *X* to follow logically from the sentences of the class $\mathfrak{K}$.[284] **[2.7.4]** On the other hand—in accord with the position which we have previously taken—[285] this condition is in general not a sufficient condition:[286] the concept of following logically[287] defined here is in fact[288] independent of the greater or lesser[289] richness[290] of the language selected as the object of investigation.[291]

[2.8 Relation to Carnap's definition] [2.8.1] Finally, it is not difficult to bring the proposed definition closer to the definition, already known to us,[292] of Carnap. **[2.8.2]** Let us agree in fact[293] to call a class of sentences $\mathfrak{Q}$[294] contradictory if it does not possess even one model;[295] analogously, one can call a class of sentences analytic if every sequence[296] is its model—[297]where one can apply both these concepts not only to whole[298] classes of sentences, but also to individual sentences. **[2.8.3]** Let us further assume that, in the language which our considerations concern, for each sentence *X* there exists[299] a negation of this sentence, i.e. a sentence *Y* such that its models are all the sequences of objects which are not models of the sentence *X*, and only these sequences.[300] **[2.8.4]** On the basis of these[301] agreements and assumptions it is easy to establish that the two cited definitions are equivalent.[302] **[2.8.5]** It is also easy to show[303] that—in accord with Carnap's intentions[304]—those and only those sentences are analytic which follow logically from every class of sentences (and, in particular, from the

279 *a sentence which follows logically from*/German: *everything that follows from nothing but*
280 *itself*/Absent in German
281 *logically*/German: *which obtains among certain sentences*
282 *the sentences among which this relation obtains*/German: *these sentences*
283 *, one*/German: *: one*
284 *the sentences of the class* $\mathfrak{K}$/German: *the class K of sentences*
285 *—in accord with the position which we have previously taken—*/Absent in German—see note 288 below
286 *a sufficient condition:*/German: *not sufficient, since*
287 *following logically*/German: *following*
288 *is in fact*/German:—*in accord with the position taken by us—is* (see note 285 above)
289 *greater or lesser*/German: *lesser or greater*
290 *richness*/German: *stock*
291 *language selected as the object of investigation*/German: *language being investigated*
292 *closer to the definition, already well known to us,*/German: *into accord with that*
293 *Let us agree in fact*/German: *For we can agree*
294 $\mathfrak{Q}$/Absent in German
295 *does not possess even one model*/German: *possesses no model*
296 *sequence*/German: *sequence of objects*
297 —/German: ,
298 *whole*/Absent in German
299 *for each sentence X there exists*/German: *to each sentence X corresponds*
300 *its models are all the sequences of objects which are not models of the sentence X, and only those sequences*/German: *it has as models those and only those sequences of objects which are not models of the sentence X (this assumption is quite essential for the Carnapian construction)*
301 *these*/German: *all these*
302 *it is easy to establish that the two cited definitions are equivalent*/German: *the equivalence of the two definitions can easily be established*
303 *It is also easy to show*/German: *Also one can show*
304 *in accord with Carnap's intentions*/German: *just as for Carnap*

empty class), on the other hand those and only those sentences are contradictory from which follow logically all the sentences of a given language.[305,J]

[3. Logical terms]

[3.1 An open question] [3.1.1] I by no means think[306] that, thanks to the above comments,[307] the problem of the construction of a correct and adequate definition[308] of the concept of following has been entirely solved;[309] on the contrary, in my opinion, a whole series of questions remains open.[310] **[3.1.2]** I would like to direct attention here to one of them,[311] perhaps the most important one.

[3.1.3] At the foundation of our whole construction lies the division of all terms of a language[312] into logical and extra-logical. **[3.1.4]** This division is certainly[313] not entirely[314] arbitrary: if we did not count among the logical terms e.g.[315] the implication sign or the quantifiers,[316] the[317] definition provided of following[318] could[319] lead to consequences manifestly contradictory to[320] everyday intuitions.[321] **[3.1.5]** On the other hand however I know no objective reasons which would allow one to draw a precise[322] dividing line between the two categories[323] of terms. **[3.1.6]** On the contrary, I have the impression that—without expressly violating everyday intuitions—one can[324] count among the logical terms also terms which logicians do not usually count among this category.[325] **[3.1.7]** The extreme would be the case in which we treated[326] all terms of the language as logical: the concept of following formally would then coincide with the concept of f o l l o w i n g m a t e r i a l l y[327]—the sentence X would[328] follow from the sentences of the class $\mathfrak{K}$[329] if and only if[330]

305 *follow logically all the sentences of a given language*/German: *every sentence follows*
306 *I by no means think*/German: *I am not at all of the opinion*
307 *, thanks to the above comments,*/German: *as a result of the consideration carried out*
308 *the construction of a correct and adequate definition*/German: *a materially adequate definition*
309 *has been entirely solved*/German: *is completely dealt with*
310 *, in my opinion, a whole series of questions remains open*/German: *: I see still several open questions*
311 *I would like to direct attention here to one of them,*/German: *I shall here point out only one of these questions—*
312 *a language*/German: *the language under consideration*
313 *This division is certainly*/German: *Certainly this division is*
314 *entirely*/German: *completely*
315 *did not count among the logical terms e.g.*/German: *e.g. wanted to count among the extra-logical terms*
316 *quantifiers*/German: *universal quantifier*
317 *the*/German: *then the*
318 *following*/German: *the concept of following*
319 *could*/German: *would*
320 *manifestly contradictory to*/German: *which manifestly contradict*
321 *everyday intuitions*/German: *common linguistic usage*
322 *precise*/German: *sharp*
323 *categories*/German: *groups*
324 *. On the contrary, I have the impression that—without expressly violating everyday intuitions—one can*/German: *; it seems to be possible to*
325 *logicians do not usually count among this category*/German: *are usually considered by logicians as extra-logical, without thereby running into consequences which would stand in sharp contrast to common linguistic usage*
326 *The extreme would be the case in which we treated*/German: *In the extreme case one could consider* (The German has 'äusseren', a misprint for 'äussersten'.)
327 *the concept of f o l l o w i n g m a t e r i a l l y*/German: *that of following materially*
328 *would*/German: *would in this case already*
329 *sentences of the class* $\mathfrak{K}$/German: *class K of sentences*
330 *if and only if*/German: *if*

either the sentence A were true or at least one sentence of the class $\mathfrak{K}$[331] were false.[K]

[3.2 Philosophical implications] [3.2.1] In order to realize the importance[332] of the question[333] under consideration from the point of view of[334] certain philosophical conceptions,[335] it is necessary[336] to direct one's attention to the fact[337] that the division of terms into logical and extra-logical exerts an essential influence on the definition also of such terms as 'analytic' or 'contradictory';[338] yet the concept of an analytic sentence[339]—in the intention of some contemporary logicians—is to be[340] a precise[341] formal correlate of the concept of *tautology* as[342] a sentence which "says nothing about the real world",[343] a concept which to me personally seems rather murky[344] but which played and still plays a prominent role in the philosophical speculations[345] of L. Wittgenstein and almost[346] the whole Vienna Circle.[L]

[3.3 Prospects] [3.3.1] Clearly, further investigations may throw a lot of light on the question[347] which interests us; perhaps one will succeed with the help of some[348] weighty arguments of an objective character in justifying the dividing line traced by tradition[349] between logical and extra-logical terms. **[3.3.2]** Personally I would not be surprised however even if the result of these investigations were to be decidedly negative[350] and if hence it would turn out to be necessary to treat[351] such concepts as following logically, analytic sentence or tautology[352] as relative concepts which must[353] be related to a definite but[354] more or less arbitrary division of the terms of a language[355] into logical and extra-logical; the arbitrariness of this division would

331 $\mathfrak{K}$: German: *K*
332 *importance*/German: *significance*
333 *question*/German: *problem*
334 *from the point of view of*/German: *for*
335 *philosophical conceptions*/German: *general philosophical views*
336 *necessary*/German: *enough*
337 *direct one's attention to the fact*/German: *note*
338 *exerts an essential influence on the definition also of such terms as 'analytic' or 'contradictory'*/German: *plays also an essential role in making precise the concept 'analytic'*
339 *yet the concept of an analytic sentence*/German: *but this concept*
340 *—in the intention of some contemporary logicians—is to be*/German: *is viewed by some logicians as*
341 *a precise*/German: *an exact*
342 *as*/German: *(i.e. of*
343 *the real world",*/German: *reality"),*
344 *murky*/German: *vague*
345 *played and still plays a prominent role in the philosophical speculations*/German: *was of fundamental significance for the philosophical considerations*
346 *almost*/Absent in German
347 *Clearly, further investigations may throw a lot of light on the question*/German: *Further research can of course clear up to a strong degree the problem*
348 *with the help of some*/German: *in finding*
349 *in justifying the dividing line traced by tradition*/German: *which will allow one to justify the traditional dividing line*
350 *Personally I would not be surprised however even if the result of these investigations were to be decidedly negative*/German: *But I definitely consider it possible that even future investigations will bring no positive results in this direction*
351 *if hence, it would turn out to be necessary to treat*/German: *that one consequently will be compelled to regard*
352 *following logically, analytic sentence or tautology*/German: *"following logically", "analytic sentence", or "tautology"*
353 *must*/German: *must each time* (The German misprints 'muss' for 'müssen')
354 *but*/Absent in German
355 *the terms of a language*/German: *terms*

be in some measure a natural reflection of that instability which can be observed in the usage of the concept of following in everyday speech.[356]

Notes

Bibliographical note. This is a summary of an address given at the International Congress of Scientific Philosophy in Paris, 1935. The article first appeared in print in Polish under the title 'O pojęciu wynikania logicznego' in *Przegląd Filozoficzny*, Vol. 39 (1936), pp. 58–68, and then in German under the title 'Über den Begriff der logischen Folgerung', *Actes du Congrès International de Philosophie Scientifique*, Vol. 7 (Actualités Scientifiques et Industrielles, Vol. 394), Paris, 1936, pp. 1–11. [Note added by Tarski in English in Tarski (1956, 1983); 'pojęciu' was misprinted as 'pojciu'.]

The present report was delivered on 16 September 1935 at a session of the First International Congress of Scientific Philosophy, which took place in Paris on 15–23 September 1935.[357]

(A) **[A.1]** I sketched[358] the example of a deductive theory that exhibits[359] the peculiarity described[360] in the year 1927 in[361] a lecture entitled: *On the consistency and completeness of the deductive sciences*, delivered at a session of[362] the Second Polish Philosophical Convention in Warsaw; there I also provided a formulation of the rule of infinite induction, which is connected in the closest way with the example discussed.[363] **[A.2]** To those[364] problems I later devoted a special article: *Einige Betrachtungen über die Begriffe der ω-Widerspruchsfreiheit und der ω-Vollständigkeit* [*Some observations on the concepts of ω-consistency and ω-completeness*], Monatshefte für Mathematik und Physik, Vol. 40, Leipzig 1933, pp. 97–112 (this article will be cited below[365] as T_1). **[A.3]** I also discuss the problems indicated quite extensively, though somewhat incidentally,[366] in my work: *Pojęcie prawdy w językach nauk dedukcyjnych* [The concept of truth in languages of the deductive sciences], Publication Series of the Warsaw Scientific Society, Section III. mathematical and physical sciences, no. 34, Warsaw 1933, cf. in particular pp. 107 ff.; the German translation of this work together with a supplement appeared

356 *; the arbitrariness of this division would be in some measure a natural reflection of that instability which can be observed in the usage of the concept of following in everyday speech*/German: *. In this compulsion the instability in the common usage of the concept of following would—at least partly—be reflected in a quite natural way*

357 *The present report was delivered on 16 September 1935 at a session of the First International Congress of Scientific Philosophy, which took place in Paris on 15–23 September 1935.*/Absent in German (the German version of this article was published in the proceedings of the aforementioned congress)

358 *sketched*/German: *provided already*

359 *which exhibits*/German: *with*

360 *described*/German: *described above as well as the formulation of the rule of inference—closely connected with it—of infinite induction already*

361 *in*/German: *, namely in*

362 *entitled*: On the consistency and completeness of the deductive sciences, *delivered at a session of*/ German: *given during*

363 *; there I also provided a formulation of the rule of infinite induction, which is connected in the closest way with the example discussed*/German: *under the title*: On the consistency and completeness of the deductive sciences

364 *those*/German: *the same*

365 *below*/German: *further below*

366 *, though somewhat incidentally.*/Absent in German

under the title:[367] *Der Wahrheitsbegriff in den formalisierten Sprachen* [*The concept of truth in formalized languages*], Studia Philosophica, vol. I, Lwów 1935, pp. 261–405 (I cite the Polish original below as T_2, the German translation as T_3).[368]

(B) **[B.1]** In connection with[369] the concept of metatheory (metascience) and with[370] the problem of the interpretation of a metatheory in the theory itself,[371] cf. e.g. T_2, pp. 18 ff., 35 and 96 ff.[372]

(C) **[C.1]** Cf. K. Gödel, *Über*[373] *formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I* [On formally undecidable propositions of Principia Mathematica and related systems I], Monatshefte für Mathematik und Physik, Vol. 38, Leipzig 1931, pp. 173–198, in particular pp. 190–191.[374]

(D) **[D.1]** In order to avoid possible reproaches, it would be necessary to limit[375] more exactly the scope of applicability of the above thesis and to[376] make more precise the logical character[377] of the rules of inference which we have in mind here,[378] in particular to describe precisely[379] what the "structuralness"[380] of these rules consists in.

(E) **[E.1]** A clear-cut opposition of the two concepts under consideration is already contained in my article[381] T_1, pp. 110 ff. **[E.2]** In contrast however to the position which I now take,[382] I expressed there a decidedly negative view[383] on the possibility of constructing an exact formal definition for the proper concept of following. **[E.3]** My position then is explained by the fact that at the time of[384] composing the cited article I did not want to use[385] any means[386] which would not be contained within[387] the framework of the theory of logical types in one of its "classical" forms; whereas one can demonstrate[388] that it is impossible to define adequately the proper concept of following using exclusively the means permitted[389] by[390] the classical theory of types—unless one considers merely

367 *Pojęcie prawdy w Językach nauk dedukcyjnych [The concept of truth in languages of the deductive sciences], Publication Series of the Warsaw Scientific Society, Section III. mathematical and physical sciences, no. 34, Warsaw 1933, cf. in particular pp. 107 ff.; the German translation of this work together with a supplement appeared under the title:*/Absent in German

368 *(I cite the Polish original below as T_2, the German translation as T_3)*/German: *(Polish Warsaw 1933; in the following cited as T_2); cf. in particular pp. 383 ff.*

369 *In connection with*/German: *On*

370 *(metascience) and with*/German: *and also on*

371 *theory itself*/German: *corresponding theory*

372 *T_2, pp. 18 ff., 35 and 96 ff.*/German: *T_2, pp. 281 ff., 301 and 370 ff.*

373 The German version misprints 'Uber' instead of 'Über;' the Polish version spells the word correctly

374 *190–191.*/German: *190 f.*

375 *reproaches, it would be necessary to limit*/German: *complaints, one should specify*

376 *above thesis and to*/German: *thesis formulated above and*

377 *character*/German: *nature*

378 *rules of inference which we have in mind here,*/German: *intended rules of inference;*

379 *to describe precisely*/German: *one should describe exactly*

380 *"structuralness"*/German: *"structural character"*

381 *my article*/Absent in German

382 *the position which I now take*/German: *my present standpoint*

383 *expressed there a decidedly negative view*/German: *expressed myself there in a decidedly negative way*

384 *of*/German: *when I was*

385 *did not want to use*/German: *wanted to avoid*

386 *means*/German: *means of construction*

387 *would not be contained within*/German: *went beyond*

388 *whereas one can demonstrate*/German: *it can however be demonstrated*

389 *permitted*/German has 'zulässlich', an apparent misprint for 'zulässig'

390 *by*/German: *in*

formalized languages of a very[391] elementary fragmentary character (precisely speaking,[392] the so-called languages of finite order; cf. T_3,[393] in particular "Nachwort [Afterword]",[394] pp. 393 ff.).

[E.4] R. Carnap in[395] his highly interesting book: *Logische Syntax der Sprache*, Schriften zur Wissenschaftlichen Weltauffassung, Band 8[396] [*The Logical Syntax of Language*, Writings on the Scientific Understanding of the World, Vol. 8], Vienna 1934 (cited below as C_1),[397] designates the old concept of following, used[398] in the construction of deductive theories, by the term 'derivability' or 'derivation' ('Ableitung'),[399] wanting[400] to distinguish it in this way from the proper concept of following ('Folge').[401] **[E.5]** Carnap extends the opposition of the two concepts to the most diverse derived concepts ('f-Begriffe'[402] and 'a-Begriffe',[403] cf. pp. 88 ff. and 124 ff.); he stresses[404] the importance of the proper concept of following and of the concepts derived from it for general theoretical considerations (cf. e.g. p. 128); finally, he defines in an exact way the concept of following for certain concrete deductive theories (cf. the next note).[405]

(F) **[F.1]** Cf. C_1, pp. 88–89,[406] and also by the same author: *Ein Gültigkeitskriterium für die Sätze der klassischen Mathematik* [*A criterion of validity for theorems of classical mathematics*], Monatshefte für Mathematik und Physik, Vol. 42, Leipzig, 1935, pp. 163–190, in particular p. 181 (cited below[407] as C_2). **[F.2]** In C_1 on[408] pp. 34 f.[409] we find[410] still another definition of following, adapted[411] to a formalized language with a very simple structure;[412] we do not cite this definition because[413] it cannot be extended in a natural way to other less elementary languages.[414] **[F.3]** Carnap also attempts[415] to define following logically not with reference to concrete formalized languages, but[416] on the basis of what he calls

391 *very*/Absent in German
392 *precisely speaking*,/German: *precisely:*
393 *cf.* T_3/German: *on this cf.* T_2
394 *'Nachwort [Afterword]'*,/Absent in German
395 *R. Carnap in*/German: *In*
396 *Schriften zur Wissenschaftlichen Weltauffassung, Band 8*/Absent in German
397 ,/German: , *R. Carnap*
398 *used*/German: *commonly used*
399 *by the term 'derivability' or 'derivation' ('Ableitung')*/German: *as* (logische) Ableitung [*(logical) derivation*] *or* Ableitbarkeit [*derivability*]
400 *wanting*/German: *in order*
401 *the proper concept of following ('Folge')*/German: *the concept of* Folge [*consequence*] as the proper concept of following
402 *f-Begriffe*/f-concepts (the letter 'f' is derived from the word 'Folge')
403 *a-Begriffe*/a-concepts (the letter 'a' is derived from the word 'Ableitung')
404 *he stresses*/German: *in doing so he stresses—as it seems to me, rightly—*
405 *; finally, he defines in an exact way the concept of following for certain concrete deductive theories (cf. the next note)*/Absent in German
406 *88-89*/German: *88 f*
407 *below*/German: *in the following*
408 C_1 *on*/German: C_1,
409 *f.*/German: *f,*
410 *we find*/German: *there is*
411 *adapted*/German: *which is adapted*
412 *with a very simple structure*/German: *of elementary character*
413 *we do not cite this definition because*/German: *this definition is not taken into account here since*
414 *extended in a natural way to other less elementary languages*/German: *carried over to languages of more complicated logical structure*
415 *also attempts*/German: *attempts*
416 *to define following logically not with reference to concrete formalized languages, but*/German: *to define the concept of following logically not only for concrete languages, but also*

'general syntax' ('allgemeine Syntax'); this attempt will be spoken about[417] below,[418] in note H.

(G) **[G.1]** About the methods and concepts of semantics, in[419] particular the concepts of truth and satisfaction, I write extensively[420] in T_2, or T_3;[421] cf. also my report: *On the establishment of scientific semantics* in this issue of 'Przegląd Filozoficzny' [Vol. 39 (1936), 50–59].[422]

(H) **[H.1]** The above remarks attack various[423] earlier attempts at defining[424] following formally undertaken by some logicians.[425] **[H.2]** They apply[426] among others—*mutatis mutandis*[427]—to the[428] definitions of following logically and of a series of derived concepts ("L-Folge" ["L-consequence"] and "L-Begriffe" ["L-concepts"]) which C a r n a p provides on the basis of his "general syntax" (cf. C_1, pp. 134 ff.): these definitions do not seem to me materially adequate[429] for just this reason, that they make the denotation of the defined concepts dependent in an essential way[430] on the richness of the language which is the object of consideration.[431] **[H.3]** Anyway, this attempt of C a r n a p's is not closely connected with the present considerations: he limits himself in fact to trying to reduce the concept of following logically to a general concept of following whose content he then does not make precise, treating it as a basic concept of "general[432] syntax".[433]

(I) **[I.1]** After the original of this paper had appeared in print, H. Scholz in his article 'Die Wissenschaftslehre Bolzanos, Eine Jahrhundert-Betrachtung', *Abhandlungen der Fries'schen Schule*, new series, Vol. 6, pp. 399–472 (see in particular p. 472, n. 58) pointed out a far-reaching analogy between this definition of consequence and the one suggested by B. Bolzano about a hundred years earlier. [Note added by Tarski in English in Tarski (1956, 1983).]

(J) **[J.1]** Cf. C_1, pp. 135 ff., in particular theorems 52.7 and 52.8; C_2, p. 182, theorems 10 and 11. **[J.2]** I will[434] take this opportunity to remark that the definition of the concept of following proposed here does not go beyond the

417 *this attempt will be spoken about*/German: *we come to speak about this attempt*
418 *below*/German: *later*
419 *, in*/German: *and in*
420 *write extensively*/German: *comment in detail*
421 *T_2, or T_3*/German: *T_2*
422 *in this issue of 'Przegląd Filozoficzny' [Vol. 39 (1936), 50–59]*/German: *Actes du Congrès International de Philosophie Scientifique, Fasc. III, Paris 1936, pp. 1–8* (The German has *0-00*.)
423 *The above remarks attack various*/ German: *By the remarks just now given some*
424 *at defining*/German: *to define the concept of*
425 *undertaken by some logicians*/German: *are attacked*
426 *They apply*/German: *These remarks can be applied*
427 *mutatis mutandis*/Absent in German
428 *the*/German: *Carnap's*
429 *"L-Begriffe"* ["*L-concepts*"]*) which Carnap provides on the basis of his "general syntax" (cf. C_1, pp. 134 ff.): these definitions do not seem to me materially adequate*/German: *"L-Begriffe"* ["*L-concepts*"]; *cf. C1, pp. 134 ff.): these definitions—insofar as they are constructed on the basis of "general syntax"—seem to me to be materially inadequate*
430 *they make the denotation of the defined concepts dependent in an essential way*/German: *the defined concepts become essentially dependent in their denotation*
431 *language which is the object of consideration*/German: *language investigated*
432 The Polish word translated 'general' has an incorrect ending, 'ogólne' instead of 'ogólnej'
433 *Anyway, this attempt of Carnap's is not closely connected with the present considerations: he limits himself in fact to trying to reduce the concept of following logically to a general concept of following whose content he then does not make precise, treating it as a basic concept of "general syntax".*] Absent in German
434 *will*/German: *would like to*

framework of the logical syntax of language[435] in Carnap's understanding (cf. e.g. C_1, pp. 6 ff.). **[J.3]** It is true that the general concept of satisfaction (or of model) is not contained within[436] the framework of syntax; we need however only a special case of this concept—the satisfaction of sentential functions in which no extra-logical constants occur,[437] and one can already characterize this special case[438] using exclusively concepts from the realm of logic and syntax.[439] **[J.4]** Between the general concept of satisfaction and this special case of it which we are using[440] there obtains almost precisely[441] the same relation as between the semantic concept of true sentence and the syntactic concept of analytic sentence.

(K) **[K.1]** It will perhaps be instructive to juxtapose the three concepts—derivability (cf. note E), following logically i.e. formally, and following materially—[442] in the special case when the class $\mathfrak{K}$[443] from which the given sentence X follows consists[444] of a finite number of sentences[445] $Y_1, Y_2, \ldots Y_n$. **[K.2]** Let us designate by the symbol "Z" the conditional sentence (the implication) whose antecedent is the conjunction of the sentences $Y_1, Y_2, \ldots Y_n$ and whose consequent is the sentence X. **[K.3]** The following equivalences can then be established:

> *The sentence X is derivable*[446] *from the sentences of the class* $\mathfrak{K}$[447] *if and only if the sentence Z is a logical thesis*[448] *(i.e. is*[449] *derivable from the axioms of logic);*
> *the sentence X follows formally*[450] *from the sentences of the class* $\mathfrak{K}$[451] *if and only if the sentence Z is analytic;*[452]
> *the sentence X follows materially*[453] *from the sentences of the class* $\mathfrak{K}$[454] *if and only if the sentence Z is true*[455].

[K.4] Of the three equivalences only the first can give rise to[456] certain objections; cf. on this matter[457] my article: *Grundzüge des Systemenkalküls (Erster Teil)*

435 *the logical syntax of language*/German: *syntax*
436 *is not contained within*/German: *cannot be fitted into*
437 *in which no extra-logical constants occur*/German: *which contain no extra-logical constants*
438 *one can already characterize this special case*/German: *this special case can already be characterized*
439 *using exclusively concepts from the realm of logic and syntax*/German: *with the help of general-logical and specifically syntactical concepts*
440 *this special case of it which we are using*/German: *the special case of this concept used here*
441 *almost precisely*/German: *roughly*
442 *concepts—derivability (cf. note E), following logically i.e. formally, and following materially*—German: *concepts: "derivation" (cf. n. E), "following formally" and "following materially"*
443 $\mathfrak{K}$: German: *K*
444 *consists*/German: *consists only*
445 *sentences*/German: *sentences:*
446 *is derivable*/German: *is (logically) derivable*
447 $\mathfrak{K}$: German: *K*
448 *a logical thesis*/German: *logically provable*
449 *is*/Absent in German
450 *follows formally*/German: *follows formally*
451 $\mathfrak{K}$: German: *K*
452 *analytic*/German: *analytic*
453 *follows materially*/German: *follows materially*
454 $\mathfrak{K}$: German: *K*
455 *true*/German: *true*
456 *give rise to*/German: *arouse*
457 *on this matter*/German: *on this*

[*Foundations of the calculus of systems (first part)*], Fundamenta Mathematicae, Vol. 25, Warsaw 1935, pp. 503–526, in particular p. 507 (brevier): **[K.5]** In connection with the above[458] theses, cf. also the works of K. A j d u k i e w i c z:[459] *Z metodologji nauk dedukcyjnych*[460] [*On the methodology of the deductive sciences*], The Publishing House of The Polish Philosophical Society in Lwów, Vol. 10,[461] Lwów 1921, p. 19, and also *Logiczne podstawy nauczania*[462] [*The Logical Foundations of Teaching*], a reprint from the *Encyklopedja Wychowania* [*Encyclopaedia of Education*],[463] Warsaw, 1934, pp. 14 and 42.

[K.6] In view of the analogy brought out between different varieties of following,[464] the question arises whether it would not be useful to introduce—besides special concepts of following—also[465] the general concept of relative character:[466] f o l l o w i n g w i t h r e s p e c t t o a c l a s s o f s e n t e n c e s[467] $\mathfrak{L}$.[468] **[K.7]** Keeping[469] the previous designations (and limiting ourselves thereby[470] to the case when[471] the class $\mathfrak{K}$[472] is finite), we would[473] define this concept in this[474] way:

> *the sentence X f o l l o w s*[475] *from the sentences of the class* $\mathfrak{K}$[476] *w i t h r e s p e c t t o t h e c l a s s o f s e n t e n c e s*[477] $\mathfrak{L}$[478] *if and only if the sentence Z belongs to the class* $\mathfrak{L}$.[479]

[K.8] Derivability would thus be[480] following with respect to the class of all logical theses,[481] following formally would be following with respect to the class of all analytic sentences, and following materially would be following with respect to the class of all true sentences.

(L) **[L.1]** Cf. L. W i t t g e n s t e i n,[482] *Tractatus logico-philosophicus*, London 1922; C_1, pp. 37–40.

458 *above*/German: *stated*
459 *K. A j d u k i e w i c z*/German: *K. Ajdukiewicz*
460 *dedukcyjnych*/German: *dedukcyjnych (On the Methodology of the Deductive Sciences, Polish)*
461 *The Publishing House of The Polish Philosophical Society in Lwów, Vol. 10,*/Absent in German
462 *nauczania*/German: *nauczania (The Logical Foundations of Teaching, Polish)*
463 *a reprint from the Encyklopedja Wychowania [Encyclopedia of Education],*/Absent in German
464 *of following*/German: *of the concept of following*
465 *—besides special concepts of following—also*/German: *besides special concepts also*
466 *character:*/German: *character, namely the concept of*
467 f o l l o w i n g w i t h r e s p e c t t o a c l a s s o f s e n t e n c e s/Italicized in German, without extra spacing
468 $\mathfrak{L}$/German: *L*
469 *Keeping*/German: *If we continue to use*
470 *designations (and limiting ourselves thereby*/German: *designation (and thereby limit ourselves*
471 *when*/German: *where*
472 $\mathfrak{K}$: German: *K*
473 *would*/German: *can*
474 *this*/German: *the following*
475 *f o l l o w s*/German: *follows*
476 $\mathfrak{K}$: German: *K*
477 *w i t h r e s p e c t t o t h e c l a s s o f s e n t e n c e s*/German: *with respect to the class of sentences*
478 $\mathfrak{L}$/German: *L*
479 $\mathfrak{L}$/German: *L*
480 *Derivability would thus be*/German: *On the basis of this definition derivability would coincide with*
481 *logical theses*/German: *logically provable sentences*
482 *L. W i t t g e n s t e i n*/German: *L. Wittgenstein*

References

Beth, E. V. 1969/1955. 'Semantic Entailment and Formal Derivability', in J. Hintikka, ed. *The Philosophy of Mathematics*. London: Oxford University Press, pp. 9–41. [Reprinted from *Mededelingen van de Koninklijke Nederlandse Akademie van Wetenschappen, Afdeling Letterkunde*, N.R. Vol. 18, No. 13, Amsterdam, 1955, 309–342.]

Carnap, R. 1934. *Logische Syntax der Sprache*, Schriften zur wissenschaftlichen Auffassung, Vol. 8, Vienna: Springer.

Carnap, R. 1937. *Logical Syntax of Language*, trans A. Smeaton, London: Routledge & Kegan Paul. [Translation of Carnap (1934) and of 22 sections not included in it for lack of space, and with additions and corrections, most suggested by Tarski.]

Carnap, R. 1963. 'Intellectual autobiography', in P. A. Schilpp, ed. *The Philosophy of Rudolf Carnap*, LaSalle, IL: Open Court, pp. 3–84.

Church, A. 1956. *Introduction to Mathematical Logic, Vol. I*, Princeton: Princeton University Press.

Etchemendy, J. 1990. *The Concept of Logical Consequence*, Cambridge, MA: Harvard University Press.

Gödel, K. 1930. 'Die Vollständigkeit der Axiome des logischen Funktionenkalküls', *Monatshefte für Mathematik und Physik* **37**, 349–360.

Gödel, K. 1931. 'Über formal unentscheidbare Sätze der Principia mathematica und verwandter Systeme I', *Monatshefte für Mathematik und Physik* **38**, 173–198.

Gómez-Torrente, M. 1996. 'Tarski on logical consequence', *Notre Dame Journal of Formal Logic* **37**, 125–151.

Gómez-Torrente, M. 1998. 'On a fallacy attributed to Tarski', *History and Philosophy of Logic*, **19**, 227–234.

Hodges, W. 1986. 'Truth in a structure', *Proceedings of the Aristotelian Society*, **85**, 135–158.

Post, E. L. 1921. 'Introduction to a general theory of elementary propositions', *Bulletin of the American Mathematical Society*, **26**, 163–185.

Quine, W. V. 1937. Review of Tarski (1936b), *Journal of Symbolic Logic* **2**, 83–84.

Sher, G. 1991. *The Bounds of Logic: A Generalized Viewpoint*, Cambridge, MA and London: MIT Press.

Sher, G. Y. 1996. 'Did Tarski Commit "Tarski's Fallacy"?', *Journal of Symbolic Logic*, **61**, 653–686.

Tarski, A. 1933a. *Pojęcie prawdy w językach nauk dedukcyjnych* [*The concept of truth in languages of the deductive sciences*], Warsaw: Nakładem Towarzystwa Naukowego Warszawskiego.

Tarski, A. 1933b. 'Einige Betrachtungen über die Begriffe der ω-Widerspruchsfreiheit und der ω-Vollständigkeit' ['Some observations on the concepts of ω-consistency and ω-completeness'], *Monatshefte für Mathematik und Physik*, **40**, 97–112.

Tarski, A. 1935. 'Der Wahrheitsbegriff in den formalisierten Sprachen' ['The concept of truth in formalized languages'], *Studia Philosophica* **1**, 261–405. [Translation of Tarski (1933a) by Tarski himself, with a 'Nachwort' ['Afterword'] (399–405) completed on 13 April 1935].

Tarski, A. 1936a. 'O pojęciu wynikania logicznego', *Przegląd Filozoficzny* **39**, 58–68.

Tarski, A. 1936b. 'Über den Begriff der logischen Folgerung', in *Actes du Congrès International de Philosophie Scientifique, Sorbonne, Paris, 1935 (Actualités Scientifiques et Industrielles 394)* (VII, 1–11), Paris: Hermann.

Tarski, A. 1936c. *O logice matematycznej i metodzie dedukcyjnej* [*On mathematical logic and deductive method*]. Lwów/Warsaw: Książnica-Atlas.

Tarski, A. 1937. *Einführung in die mathematische Logik und in die Methodologie der Mathematik* [*Introduction to mathematical logic and to the methodology of mathematics*]. Vienna: Springer. [An 'exact translation' (Tarski 1946/1941: xi) by Tarski of Tarski (1936c).]

Tarski, A. 1944. 'The semantic conception of truth and the foundations of semantics', *Philosophy and Phenomenological Research* **4**, 341–375.

Tarski, A. 1946/1941. *Introduction to Logic and to the Methodology of Deductive Sciences*, 2nd edn. New York: Oxford University Press. [First edition published 1941. Translation, with additions, of Tarski (1937).]

Tarski, A. 1956. *Logic, Semantics, Metamathematics. Papers from 1923 to 1938*, trans. J. H. Woodger. Oxford: Clarendon Press.

Tarski, A. 1969. 'Truth and proof', *L'Age de la science* **1**, 279–301.

Tarski, A. 1983. *Logic, Semantics, Metamathematics. Papers from 1923 to 1938*, 2nd edn, trans. J. H. Woodger, ed. and intro. John Corcoran. Indianapolis, IN: Hackett.

Tarski, A. 1986/1966. 'What are logical notions?', *History and Philosophy of Logic*, **7**, 143–154. [Delivered as a lecture in 1966 and 1973, and subsequently edited by John Corcoran.]

Tarski, A. 1987/1944. 'A philosophical letter of Alfred Tarski', *The Journal of Philosophy*, **84**, 28–32. [A letter to Morton White, dated 23 September 1944.]

Tarski, J. 2000. Personal communication.